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Exploring The Stages of Polya's Problem-solving Model during Collaborative Learning: A Case of Fractions

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ABSTRACT This paper reports on a small-scale action research, which investigated the processes involved in problem-solving in a mathematics class. Grade ten learners (*n = 47*) at a South African middle school were involved in the study. The participants from two classes attempted the solution of tasks involving the fraction concept. In one class, the learners worked in groups and in the other class, the learners worked individually. A qualitative method was adopted for data capture and analysis. Social constructivism was adopted as a theoretical framework and the stages advocated by Polya were interrogated when analyzing the learner responses on their problem solutions. The results revealed that those learners working in groups demonstrated most of the stages of the Polya linear problem-solving model. The findings helped identify which stages of the model promote effective problem-solving and some recommendations are made for classroom practitioners engaging their learners in problem-solving.

INTRODUCTION

Many studies (Brijlall 2014c; Maharajh et al. 2008; Schoenfeld 1992) have investigated the successes of collaborative learning. These studies point to improved higher order learning abilities as a consequence of collaborative learning. The question not frequently explored is what mental mechanisms are being put in place to enforce such improved learning abilities. Hence, this study delved into identifying the rationale for these improved learning skills by reflecting on a problem-solving model introduced by mathematician George Polya. This study was conducted within an Outcome Based Education (OBE) paradigm. Outcome based education formed the foundation of the current curriculum in South Africa. Despite an envisaged revamp of OBE, certain principles still remain. For example, that it is encouraged that a learner-centered and activity based approach to education be emphasized for the learners to learn effectively in mathematics (DoBE 2010), will remain as a principle. Learners should be able to reflect on and explore a variety of strategies. They should also be culturally and aesthetically sensitive across a range of social contexts. However, studies like Tobias (2006) have provided evidence to indicate that students demonstrate a sense of being mathematically helpless when it comes to word problems. Most of such investigations allowed learners to solve problems on an individual basis. This study however decided to look at collaborative learning. Other studies like Brijlall (2008, 2011) and Maharajh et al. (2008) explored collaborative learning in multilingual, high school classrooms and tertiary students, respectively. The teachers' views on practical work, whilst working with fractions, were investigated by Maharaj et al. (2007). The findings in those studies revealed that collaborative learning benefitted mathematical learning. However, studies involving grade ten learners were not carried out by these researchers and this paper reports on such an investigation. Furthermore, this paper explores a possible relationship between successful problem-solving and the levels of Polya's model.

The learning of mathematics can be fun and enjoyable if taught in a familiar context related to the culture of the learners. Difficulty in problem-solving in mathematics arises when a context is unfamiliar to them (Brijlall 2011). They should therefore be provided with problem tasks, which have given information familiar to their life experiences. For this reason the two tasks provided comprised information the learners could relate to. Also, the problem-solving skills should be entrenched in the minds of the learners for use during later years (Sun 2011). It was shown by Loji (2012) that a higher order thinking needs to be developed early to enhance problem-solving skills and discovery of new knowledge.

Constructivism

According to constructivist Von Glasersfeld (1987), constructivism is seen as a cognitive theory of learning. It is emphasized that knowing is thought of as knowledge constructed by the learner as a process of adaptation based on the learners experience and constantly modified by the learner's experience. Thus in constructivism the learner does not discover an independent, pre-existing world outside his/her mind. There seems to be an implication that a learner cannot know, say a certain mathematical concept unless it is done through their experience, and the learner can only know what they have constructed and modified according to their further experience. According to this theory, knowledge is viewed as fitting with experience, so that if the experience changes the knowledge also get modified. It is therefore important that the learners are encouraged to talk about their thoughts to each other and to the teacher. To talk about what one is doing, ensures that one is examining it. In this study the result of such examinations led the learners to discuss their views of the problem and their own tentative approaches.

A constructivist sees the individual as trying to make sense of his/her experiences with the intention that the learners construct their own meaningful methods. Thus, much of cognition is problem-solving, while little of what typically occurs in a school classroom could be considered problem-solving because the learner is rarely allowed to make decisions (Grayson 1996). In the constructivist approach, learners are assumed to construct their own mathematical conceptual understanding as they take part in cultural practices, and whilst they interact with each other. Inherent in this cultural perspective is the notion that students must have a point of view that exists as a central aspect of knowledge (Schoenfeld 1992: 340). According to the constructivists, teachers also need to reflect on the developmental progression of thought processes thinking to understand the wide range of thinking patterns of students in class and to plan task for groups and individuals. In so doing, they move beyond traditional teaching and become curriculum builders (Steffe 1991). This means that both teachers and learners become more responsible for learning. Teachers require not only to have a deep and thorough pedagogical content knowledge (Bansilal et al. 2014; Brijlall 2011, 2014a; Brijlall and Isaac 2011; Brijlall and Maharaj 2014, 2015) but it is also imperative that mathematics teachers adapt learning design to focus on better learner understanding. The researchers hence, decided to approach problem-solving, by allowing learners to interact with each other and communicate their ideas to encourage the solution of the given problems. Within groups, peer engagement necessitated a social situated for collective decision-making. The two problem tasks chosen were based on knowledge of fractions and fraction manipulations. This topic was chosen as Maharaj et al. (2007) found that most learners struggle with fractions in middle school mathematics.

Collaborative Learning

Researchers are constantly experimenting with ways to employ to achieve success in the teaching and learning of mathematics. The teacher plays a pivotal role in this regard. For instance, Ramnarain (2011) showed that the teachers used questioning to support learning in science. The researchers suggest that learners can derive benefit from working with peers. Barkley et al. (2005: 4) defined collaborative as to work with another or others. This means that students are working in groups to achieve shared learning goals. They also called this learning as cooperative learning, team learning, group learning, or peer-assisted learning. Cooperative learning requires students to work together on a common task, sharing information and supporting one another (Barwell 2003; Brijlall 2014b). This project formulated the following research question based on the principles fostered in outcomes based education (OBE) and collaborative learning: *How do learners engage the processes postulated by Polya when solving mathematics word problems during collaborative learning*? In pursuing this question, the researchers emphasize that that did not intend to compare collaborative learning with individual learning but to explore the prevalence of the stages of Polya's linear problem solving model (PLM) in each case.

Knowledge Acquiring Perception of the Learner

Constructivists indicate that knowledge is formed as a learner acts upon a problem. Stears and Gopal (2010) note that the knowledge chil-

dren bring to the classroom may find expression in a variety of activities. The activity in this project involved the solving of word problems. The two tasks were non-routine problems because learners had not seen them during the teaching hours, but were supposed to be solved using their previous experience or their pre-existing knowledge relating them to their real life situations. Since the school was located in a rural area, learners were familiar with the animals that they were given to work with, even Task 2 dealt with familiar concepts of fractions, which were dealt with earlier in class. The following word problems were presented to learners to solve:

Task 1 On a farm I have goats and chickens. My son counted 70 heads and my daughter counted 200 legs. How many chickens and goats do I have?

Task 2A research surveying marriages involved adult couples above the ages of 18. The fraction of men in this population who are married is 3/5. What fraction of the population is single women?

The abstractions made are constructed by the learner and the learners discover the properties of the solution from the action. This discovery is a construction by a learner from what he or she already knows. In mathematics this becomes more evident because learners are just introduced to a problem like the one of calculating the number of goats and chicken. How they solve this problem relies on their pre-knowledge and previous experiences in problem-solving in mathematics.

The researchers observed that the students were very interested and involved in the activity. The high involvement of the student is illustrated by the fact that some of them having finished Task 1 started by trying some other possibilities such as drawing 70 heads. They started to put the required legs on the diagram used. One of the aims of teaching problem-solving is for learners to communicate using their own meaningful methods (Von Glasersfeld 1987). The grade ten syllabus also requires learners to identify, solve problems and make decisions using critical and creative thinking. Grade ten learners are required to work effectively with others as members of a team, group organization and community (DOE 2003). The researchers explored whether the stages of Polya (1957, 1975) were implemented. Polya suggested that problem-solving should follow the following stages: (a) understanding the problem, (b) devising a plan, (c) carrying out the plan, and d) looking back. Each level (which Polya called principle) is summarized below:

1) First Stage of PLM

Understanding the problem: This seems so obvious that it is often not even mentioned, yet learners are often hindered in their efforts to solve problems simply because they do not understand it fully or even in part.

2) Second Stage of PLM

Devising a plan: He mentioned that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems

3) Third Stage of PLM

Carrying out the plan: This step is usually easier than devising a plan. In general, all one needs is care and patience, given that one has the necessary skills. Persist with the plan that has been chosen. If it continues not to work discard it and choose another. One should not be misled, this is how mathematics is done, even by professionals.

4) Fourth Stage of PLM

Looking back: He mentioned that much can be gained by taking the time to react and look back at what has been done, what worked, and what did not. Doing this will enable one to predict what strategy to use to solve future problems.

RESEARCH METHODOLOGY

A qualitative research method was adopted and focused on the teacher as a researcher. This focus of enquiry enhanced the action research carried out in this study. The qualitative paradigm was employed, as the investigation was not concerned with statistical accuracy, but with detailed and in-depth analysis. The main focus of the paper was to explore the learners' experiences during the mathematics class in the problem-solving tasks. The data, which was captured

to answer the research question, was suitable for a qualitative treatment. The main participants in the study were 24 grade ten learners. This is a sample out of 47 learners in a grade ten classroom. The 24 learners consisted of 12 boys and 12 girls whose ages ranged from 14 to 17 years. The 24 learners were divided into 6 groups of 4 and 23 were working individually. Semi-structured questionnaires were used to establish what transpired in and out of the problem-solving class during the research period by grade ten learners. Learner conversations were audio recorded. The researcher reflected on the classroom activity and deciding on what could inform future action. Discussions were held with the learners by the observer and the responses of learners, expressions of the views on what was observed, and how these reflections could inform pedagogy were recorded.

RESULTS AND DISCUSSION

Comparison in Achievement Between the Two Classes

For the sake of the interview, twelve learners were chosen, six from each class. The researchers decided to choose the individual learners with about the same academic achievement as the group leaders. In this way, they assumed that the contribution made by the group learners to the written responses would be similar to the contribution made by the individual learners. Their March term score for mathematics were used. Table 1 shows the match of scores.

Table 1: Scores for March test

Group leader (%)	*Individual (%)*
A-73	1-74
B-68	2-68
C-63	3-60
D-51	4-48
E-40	5-39
F-34	6-36

Learners' General Information

The researchers recorded general information including gender and language of the two groups of learners. Hossain and Tarmizi (2012) explored gender-related effects of group learning in mathematics. Despite the fact that the researchers were not focusing on gender issues in this study, they thought it interesting to note such details to better understand the background of the participants. All participants were second language English speaking. Tables 2 and 3 show this information.

The Teaching of Problem-solving in Grade 10 Class

Participants were asked to construct their own meaningful methods to solve problems. In order to check on these steps, the researchers proceeded with the analysis of their solutions by focusing on the following questions:

(1) Which stages of PLM are demonstrated in the learners written solutions?
(2) How was the mathematical correctness of the solutions aided by these stages of PLM?

Table 2: Group information

Groups	*A*	*B*	*C*	*D*	*E*	*F*
Age	14	15	15	16	16	16
Gender	F	F	F	M	F	F
Home Language	IsiZulu	IsiZulu	IsiXhosa	IsiZulu	IsiZulu	IsiZulu

Table 3: Individuals information

Individual	*1*	*2*	*3*	*4*	*5*	*6*
Age	16	16	17	14	15	16
Gender	F	M	M	F	F	F
Home Language	IsiZulu	IsiZulu	IsiZulu	IsiZulu	IsiZulu	IsiZulu

Instructions were specified by the researcher to the learner when they commenced the task. They were asked to read the information given in each question and answer both tasks. During this time the researcher walked around observing learner engagement. The researcher found that learners were actively involved in the activity, they were communicating with each other in the groups, debating about their solutions until they reached the solution. While they were busy sharing ideas and information, the group leader was jotting down what was supposed to be presented to the class.

Learner's Responses

The written responses of the twelve learners were characterized by the following codes:

a. **Correct responses** – Correct mathematical ideas used and got task correct.
b. **Incorrect responses** – Task done but nothing correct.
c. **Completely cannot respond**- No work done but the blank answer sheet submitted.
d. **Partially correct responses**- Some steps done correctly.

The group responses are summarized in Table 4.

The effect of collaborative learning seemed to impact positively in problem-solving. This the researchers deduce since: 1) fifty percent of each task was successfully solved, 2) all groups provided a response, 3) only seventeen percent proved unsuccessful, and 4) twenty five percent of the learners could provide partial responses, which could mean that the mathematical learning process is taking place.

Group A's responses were correct for both tasks. The written responses possessed features that indicated all four stages of PLM. In their responses it was clear that they understood the problems, devised a plan, carried out the plan and lastly they looked back. This group first wrote: "*If there are 70 heads of goats and chicken and 200 legs*" at the beginning. They then drew diagrams as follows: (Fig.1)

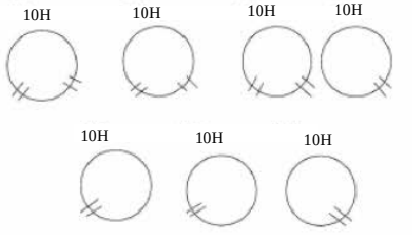

Fig.1. Using diagrams as a problem solving strategy for task 1

This represented a mechanism to assist them when devising a plan. They wrote: "*The first three H have 120 legs where each head contains 4 legs for a goat. The last four H contain 80 legs where each head contains 2 legs for a chicken. Now for* 3H⇒30 heads and 4H⇒40 heads." This was actually their explanation to the solution of the problem in carrying out the plan. In the end they wrote: "*There are 30 goats and 40 chickens. So 30 + 40 = 70 and 120 + 80 = 200*." This showed that they looked back at their findings satisfying the given information. The success in this solution highlights that all Polya's four stages were prevalent during the attainment of the correct answer. From this the researchers can induce a possible didactic cue to implement in their teaching. The researchers could probably ask the learners to actually use these levels as a checklist when solving mathematics problems. The researchers look at Group A's written response to Task 2 in Figure 2. Firstly, Group A seemed to have understood the statement of the problem in Task 2. This the researchers can assume as they wrote: *"3/5 of 100 men is 60"*, which implied that they considered an equal split of the couples. Secondly, the plan used by them

Table 4: Group responses

	Correct responses		*Incorrect responses*		*Completely cannot respond*		*Partially correct*	
	Task 1	*Task 2*	*Task 1*	*Task 2*	*Task 1*	*Task 2*	*Task 1*	*Task 2*
	A	A		D			C	
	B	B					F	F
	D	C		E			E	
Total	3	3	0	2	0	0	3	1

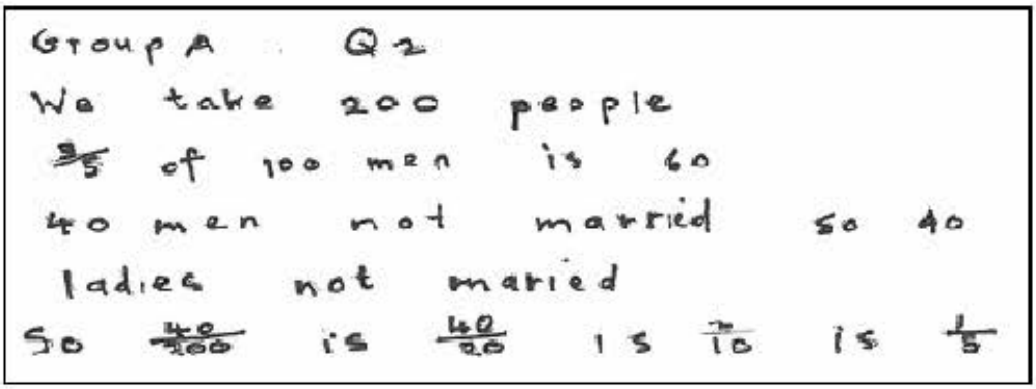

Fig. 2. An example of a correct written response for Task 2

was an instrumental one based on mental reasoning. They did not, however, look back at the question. This would suggest that the first three stages of PLM are partially responsible for successful problem-solving.

Group C displayed partial responses to Task 1. They understood the problem, and devised a plan, which could have worked. They did not satisfy stage four in Task 1. The first three levels were implemented by this group in Task 2 as can be seen in Figure 3. The researchers may assume that this group understood the problem statement for Task 2. The plan they devised involved the use of diagrams and flowcharts. This strategy helped visualize the solution in a holistic manner. However, they did not look back at the problem. For Task 2, Group D and Group E provided incorrect solutions. They failed to devise a legitimate plan and invariably were on the wrong path. Generally correct solutions displayed the first three stages of PLM. However, the converse of this, namely the first three stages of PLM is no guarantee to completely correct solutions to mathematics problems. The analysis of this data showed that the fourth stage is an absolute necessity for problem solving.

Comments on Polya's Stages

Group A had drawn pictures in order to solve problem task one. This concurs with the stages of PLM as Polya (1975) stated that the drawing

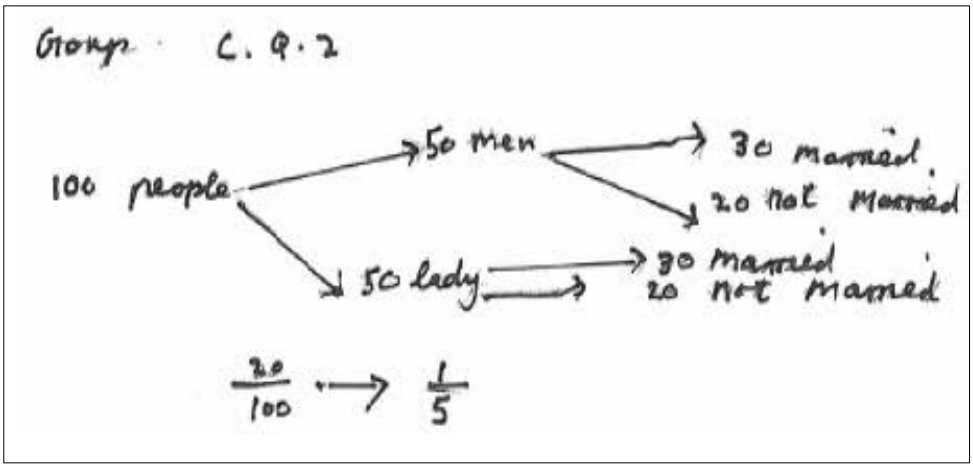

Fig. 3. A correct written response by Group C for Task 2

of pictures was an important aspect in discovery and problem-solving. These stages implicitly helped learners cope with problem-solving in this context.

The successful learners looked at the question with understanding, devised a plan, communicated how to carry out plan and lastly, in one case, they looked back. These stages surfaced during the collaborative learning despite the fact that these learners were not taught about the stages of PLM. Although the individual participants displayed some features of the PLM, nobody could solve Task 2 correctly (see Table 5).

The written response of Individual 1 had displayed the four stages of PLM. She understood the question and devised a plan where she had guessed and got the correct answer. She carried out her plan and lastly looked back. In Task 2- the problem has been done but she lacked basic operational (multiplication) skills. The researchers found that the written response for task one by Individual 2 had displayed three stages of PLM. She failed to do Task 2 because it seemed that she did not understand the question. Individual 3 did not understand both tasks. His responses were both incorrect. In his case, the researchers could assume that he had no clues as he had no help from his peers. Individual 4 had not responded at all. It seemed that he had no understanding of the demands of the problem. Individual 5 had no idea on how to solve task 1 but tried task 2 where she got both fractions correct and failed to carry out the plan. Individual 6 did not understand the task. Both his responses were incorrect.

CONCLUSION

This paper explored how learners engaged the processes postulated by Polya when solving mathematics word problems during collaborative learning. In pursuing this exploration the researchers emphasize that they did not intend to compare collaborative learning with individual learning but to explore the prevalence of the stages of Polya's linear problem-solving model (PLM) in each case.

A qualitative method was adopted in data capture and analysis. Social constructivism was adopted as a theoretical framework and the stages advocated by Polya were interrogated when analyzing the learner responses on their problem solutions. The results revealed that those learners working in groups demonstrated most of the stages of the Polya linear problem-solving model. The findings helped identify which stages of the model promoted effective problem solving and make recommendations to mathematics classroom practitioners who engage their learners in problem-solving.

RECOMMENDATIONS

The situation of very low performance in problem-solving has to be overcome by both mathematics teachers and learners. This can be accomplished in various ways as follows.

a. Development and use of mathematical skills displayed by learners from other experiences to enhance appreciation and their attitude towards problem-solving. This study highlighted this by using tasks, which had information from everyday life experiences.
b. Learners should be regarded and accepted as intelligent and creative individuals whose questions are valued. Teachers should therefore afford the learners more choice in discussion. This would give the learners effective decision-making powers in problem-solving. This can be done by allowing them to work collaboratively. The stages of PLM could assist directly

Table 5: Individual responses

	Correct responses		*Incorrect responses*		*Completely cannot respond*		*Partially correct*	
	Task 1	*Task 2*	*Task 1*	*Task 2*	*Task 1*	*Task 2*	*Task 1*	*Task 2*
	1			2	4	4		1
	2		6	6				
			3	3				
			5					
Total	2	0	3	3	1	1	0	2

this discussion. Also, they need to negotiate their solutions with their peers in class and be allowed opportunity to take ownership of their learning.

c. Learners should be allowed to use any language to communicate mathematical ideas, concepts, generalizations and thought processes. This will include the use of language to express mathematics investigations and interpretation.

d. Teachers must introduce and present problem-solving in an interesting and joyful manner. OBE has introduced a new thinking in education. A suggestion from this research is to employ and instill the stages of Polya's problem-solving model. Mathematics teachers should instill in their learners the need of a checklist, which can be adopted as a problem-solving teaching-learning activity during problem-solving in class.

LIMITATIONS

This study was a case study and only considered tasks involving fractions. The results could not be generalized. There is scope for further research in this area involving larger scale action research to determine the effectiveness of PLM in mathematics problem-solving. Research involving other mathematics topics should be explored.

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