Finite Element Design and Multi-objective Optimization of Four Pole Reluctance Motor Based on NSGA-II Intelligent Algorithm

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***Abstract*—The design of a four-pole reluctance motor with multiple objectives is discussed in this paper using a finite element design methodology based on multi-objective genetic algorithm. Non-dominated genetic algorithm (NSGA-II) is used because of its high performance and intensification in optimization problems. The global sensitivity chart revealed that the motor's stator pole embrace and yoke thickness are key parameters for the optimization objectives, while the rotor's pole embrace should be restrained and closely associated with these two key parameters. According to the optimization and sensitivity analysis results, a final design which is superior to the base design was achieved. There were 15 % and 13.2 % improvement in the optimized model in terms of the average torque and efficiency respectively. Also, the optimized model recorded a reduction in the average torque ripple and total loss by 1.55 % and 30.1 % respectively. This demonstrates the NSGA-II intelligent optimization program is a suitable framework to optimize specified objective functions.**

Keywords*—average torque, multi-objective functions, pole embrace, optimization, reluctance motor, sensitivity

I. Introduction

Electric motors are the prime movers of industry. The optimum design of these motors is therefore imperative for productivity and economic growth. Hence, the need for a design that optimizes several objective functions such as efficiency, average torque and ripple, torque per rotor volume, torque per weight, and the noise, are desirable [1].

Switched reluctance motors (SRM) have become an attractive option for various industrial applications such as electrical vehicles, wind generators, flywheel energy storage, aeronautics, extractors, air-conditioners, shipbuilding, machines tools, centrifugations [2-5]. Thus, SRM is a formidable contender to replace traditional machines in future manufacturing. Its industrial prospects are particularly promising because, while SRM technology is relatively simple to manufacture, alternative contenders such as permanent magnet machines are more difficult to produce [6, 7]. However, SRM has high torque ripples associated with its performance. In typical applications of SRM where improved torque and efficiency, and minimized torque ripple are expected, the desired objectives cannot be represented by a single based objective function; rather, it is a multi-objective optimisation problem, despite the fact that different objective functions often result in conflicting design requirements [8]. As a result, a satisfactory solution must be found in order to meet all performance objectives.

Genetic algorithm (GA) has been successfully applied to various optimization problems which has been reported in several research works [9, 10]. In [11], the NSGA-II was used to decide the best pole shape design for improving SRM performance. [12] discusses the use of a Pareto archived evolution strategy based multi-objective optimisation to enhance SRM torque of SRM. [13] proposes a rigorous system for multi-objective SRM design optimization based on a combination of design of experiments and particle swarm optimization. A genetic fuzzy algorithm for SRM design was proposed in [14]. [15] used a Latin hypercube sampling method to optimise the switched reluctance generator from the standpoint of increasing its performance and reducing the motor's volume. For rapid and accurate SRM optimisation, an improved reduced order computational system of flux tubes was introduced in [16]. In [17], a non-elitist multi-objective genetic algorithm-based design optimisation was addressed to enhance the torque and efficiency of SRM. Different intelligent algorithms were used in these references for multi-objective optimisation of various devices, and good results were obtained. However, the effect of rotor and stator pole embrace (ratio of pole arc to pole pitch) on the performance of the motor has not been considered in the multi-criteria optimisation process, hence, this study for improvement.

The aim of this study is to optimize three geometrical parameters of a doubly salient 3-phase 6/4 industry SRM to improve the average torque and efficiency, minimize average total loss and torque ripples. The contribution of this work are: optimization of dimensionless parameters; coupling Maxwell software to Ansys DesignXplorer under ANSYS workbench environment; implementation of combined effects of design of experiments, response surface, sensitivity analysis, and intelligent algorithm (NSGA-II) based optimization of the motor to avoid degradation of one parameter as a result of improvement of another requirement.

II. Mathematical Modelling of SRM

The stator of SRM has three pole pairs, carrying the three motor windings, and the rotor has several nonmagnetic poles. SRM produces torque by energizing a stator pole pair, inducing a force on the closest rotor poles and pulling them toward alignment.

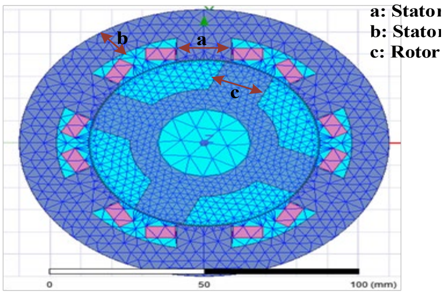

Fig. 1. Meshed model of the studied motor

TABLE I. PARAMETERS OF THE INDUSTRY MOTOR

Part	Parameter	Values
Design restriction	Rated voltage	380 V
	Rated speed	1500 rpm
	Power output	1.5 kW
Stator	Pole number	6
	Outer diameter	120 mm
	Inner diameter	75 mm
	Pole embrace	0.45
	Yoke thickness	12 mm
Rotor	Pole number	4
	Pole embrace	0.3
	Yoke thickness	9 mm
	Inner diameter	30 m

TABLE II. VARIATION RANGES OF THE PARAMETERS

Parameters	Variables	Range
Stator pole embrace, Es	P1	0.25 – 0.500
Stator yoke thickness, Ys	P2	9.00–13.00 mm
Rotor pole embrace, Er	P3	0.25 – 0.500
Avg. Torque, Tavg.	P4	4.86 Nm
Avg. Torque Ripple, T.R	P5	3.23
Efficiency, Eff.	P6	85 %
Avg. Total Loss, Tloss	P7	53.8 W

Due to the effect of magnetic saturation on the flux linkage-to-angle, λ(θph) curve, the mathematical model of SRM is highly nonlinear. The instantaneous torque for the 3-phase motor adopted in this work is derived from the phase voltage equation as:

$$v_{ph} = R_s i_{ph} + \frac{d\lambda_{ph}(i_{ph},\theta_{ph})}{dt} \tag{1}$$

$$L_T(i_{ph}\theta_{ph}) = \frac{d\lambda_{ph}(i_{ph},\theta_{ph})}{di_{ph}} \tag{2}$$

$$E_{ph} = \frac{\partial\lambda_{ph}}{\partial\theta_{ph}}\omega \tag{3}$$

$$v_{ph} = R_s i_{ph} + L_T(i_{ph}\theta_{ph})\frac{di_{ph}}{dt} + E_{ph} \tag{4}$$

$$W(i_{ph},\theta_{ph}) = \int_0^{i_{ph}} \lambda_{ph}(i_{ph},\theta_{ph})di_{ph} \tag{5}$$

$$T_{ph} = \frac{\partial W(\theta_{ph})}{\partial\theta_r} \tag{6}$$

$$T_{ph}(i_{ph},\theta_{ph}) = \int_0^{i_{ph}} \frac{\lambda_{ph}(i_{ph},\theta_{ph})di_{ph}}{\partial\theta_{ph}} \tag{7}$$

The equation for motion is given as:

$$J\frac{d\omega}{dt} = T - T_L - B_m\omega \tag{8}$$

Thus, the instantaneous torque is:

$$T = \sum_{j=1}^{3} T_{ph}(j) \tag{9}$$

The torque ripple of the motor is given as:

$$T_{ripple}\% = \frac{T_{peak} - T_{\min}}{T_{peak}} \times 100 \tag{10}$$

The motor efficiency is defined as:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{T_m \cdot \omega}{V_{dc} \cdot I_{dc}} \tag{11}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \tag{12}$$

$$P_{loss} = P_{cu} + P_{fe} + P_{mech.} \tag{13}$$

θ_{ph} is the angle per phase; i_{ph} is the current per phase; R_s is the stator resistance per phase; λ_{ph} is the flux linkage per phase; V_{ph} is the voltage per phase, J is the rotor inertia; ω is the mechanical rotational speed; T is the rotor torque; T_L is the load torque; J is the rotor inertia; B_m is the rotor damping; T_{peak} is maximum torque; T_{min} is torque of intersection of two curves; T_m is average torque; P_{cu} is the copper loss; P_{fe} is the iron loss which depends on the level of magnetization and excitation frequency.

A. The SRM Model

In this study, a 6/4 SRM as shown in Fig. 1 is used as the design case and was designed at the speed of 1,500 rpm, power of 1.5 kW and voltage of 380 VAC. The initial parameters of the motor are shown in Table I. 2D Finite element (FE) modelling of the motor was chosen because of its accuracy to model complex geometry and high computation time involved in the 3D modelling. The Ansys Maxwell 2D package software was used because it allows the motor geometry to be parameterized and to create different output variables (average torque, T.R, efficiency, losses) by means of a visual basic script.

III. OPTIMIZATION APPROACH

Optimization is a search problem that seeks better objectives of a function. A minimum of two factors are required to perform the multi-objective task of the studied motor accurately. First, the mathematical model of the motor (as shown in section II) which gives the various motor characteristics such as average torque and efficiency for any set of variable parameters. The search algorithm is the second factor which works on the principle of multi-attribute rule.

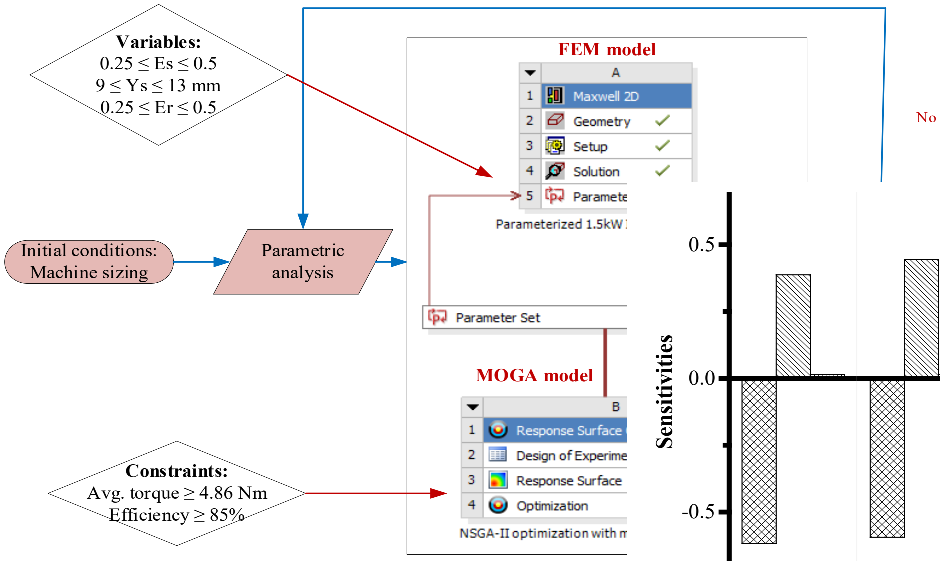

Fig. 2. Illustration of the optimization process

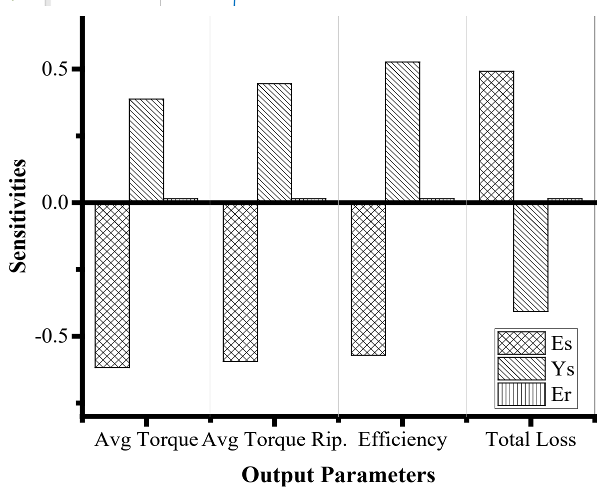

Fig. 3. Sensitivity plot of the optimization results

The details of the search algorithm are explained in Fig. 2 while Table II shows the range of variable parameters. The ANSYS Workbench framework was used to incorporate the 2D FE model into the main optimization program. After the selection of three parameters (stator embrace and yoke thickness, and rotor embrace) as the input variables, and four parameters (average torque, efficiency, torque ripple, losses) as the output variables in the first step, the second step was the multi-objective optimization to search for the best solution of machine sizing under certain constraints and objectives.

The multi-objective optimisation problem can be stated as [18]:

$$\begin{cases} Minimize\ \vec{F}(\vec{X}) \\ subject\ to: \\ \\ \vec{g}(\vec{X}) \le 0 \\ and\ \vec{h}(\vec{X}) = 0 \\ \\ \vec{X} = \{x_1, \ldots, x_n\} \end{cases} \tag{14}$$

The vector $\vec{F}(\vec{X})$ includes several objective functions, the goal is to seek to minimize (or maximize) the objective functions that are sometimes contradictory, since reducing one goal leads to an increase in another, so the solution is often a compromise between these objectives [19], thus, there is need for intelligent algorithm which will help the designer to select the best candidate without sacrificing the other parameters of the motor

A. NSGA-II for SRM Design Optimization

The optimization process is carried out using MOGA method (Multi-objective Genetic Algorithm). It is a variant of NSGA-II (Non-dominated Sorted Genetic Algorithm-II) based on controlled elitism concepts [19]. It supports various goals and constrains when aiming for a global maximum, making it ideal for the purpose of this study. The concept of NSGA-II is to produce the design space points of *M* populations at random [20]. In a model vector *M* called a chromosome, the system is discretized into *P* parameters. According to the natural terms of genetic theory, each parameter m_j, ($j = 1 \ldots P$) is referred to as a gene. A gene can be classified as a binary encoding of a parameter given by [21]:

$$m_J = m_j^{\min} + \frac{(m_j^{\max} - m_j^{\max})}{2^n - 1} \cdot \sum_{i=0}^{n-1} b_i 2^i \tag{15}$$

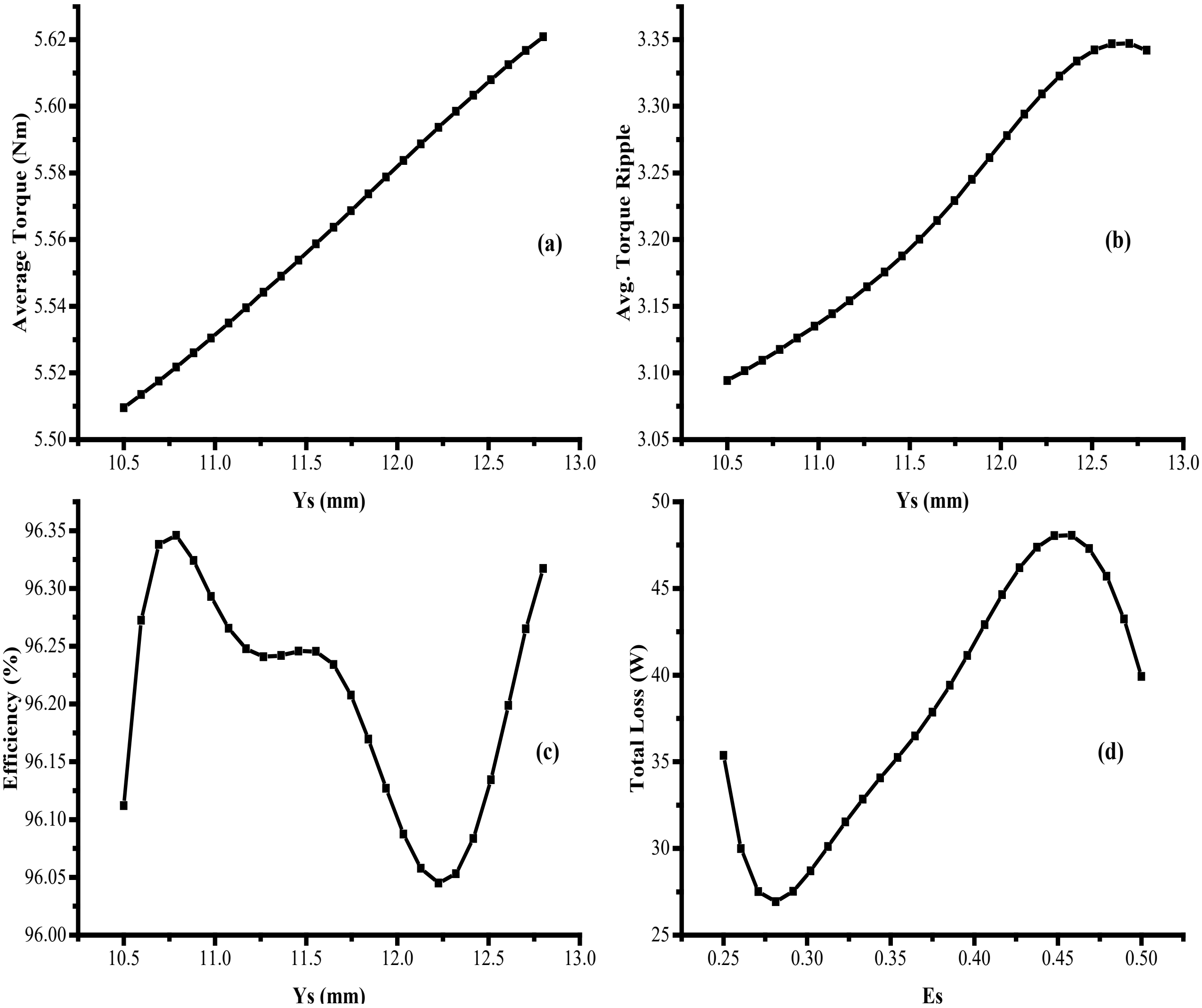

Fig. 4. Response points plots of the motor's parameters
(a) Avg. Torq./Ys (b) Avg. Torq. Rip./Ys (c) Eff/Ys (d) Loss/Es

The n-bit string of the binary representation of m_j is $b1$, $b2$… bn-1, and m_j^{min} and m_j^{max} are the minimum and maximum admissible values for m_j, respectively. These individuals' genes are combined in meaningful ways to create new solutions, which are then analysed and ranked using an objective function value.

After non-dominated sorting, the genetic algorithm's three basic operations of selection, crossing, and mutation yield the first generation of progenies. The selection's role is to pick individuals from the population based on their fitness. Secondly, starting with the second generation, the parent and progeny populations are merged and non-dominated sorted quickly using the equations below:

$$\text{Offspring1} = a*\text{Parent1} + (1-a)*\text{Parent2} \tag{16}$$

$$\text{Offspring2} = (1-a)*\text{Parent1} + a*\text{Parent2} \tag{17}$$

Finally, the genetic algorithm's basic procedure generate a new progeny population. A polynomial mutation operator is used to enforce mutation for continuous parameters as depicted in (18)

$$C = P + (\text{Upper Bound} - \text{Lower Bound})\delta \tag{18}$$

Where C is the child, P is the parent, and δ is a small variance that can be determined using a polynomial distribution. The new population would gradually contain better chromosomes (best individuals or parameters) and will eventually converge to an optimal population with the best chromosomes [19].

IV. Optimization Results And Discussions

For the solution of this study, the NSGA-II algorithm is set for 2000 estimated number of evaluations with 100 numbers of samples per iteration. The maximum allowable pareto percentage is 70, while the maximum number of candidates is 3. The Pareto optimal frontier approach is the multi-objective optimization design method used in this study. This method generates a set of optimal solutions that satisfy the design requirements and the NSGA-II algorithm optimized distribution map of the motor output. According to the motor's design specifications, the average torque is the most important factor, followed by the efficiency, ripple and the total loss. Therefore, the final selected candidate will have improved average torque reliability.

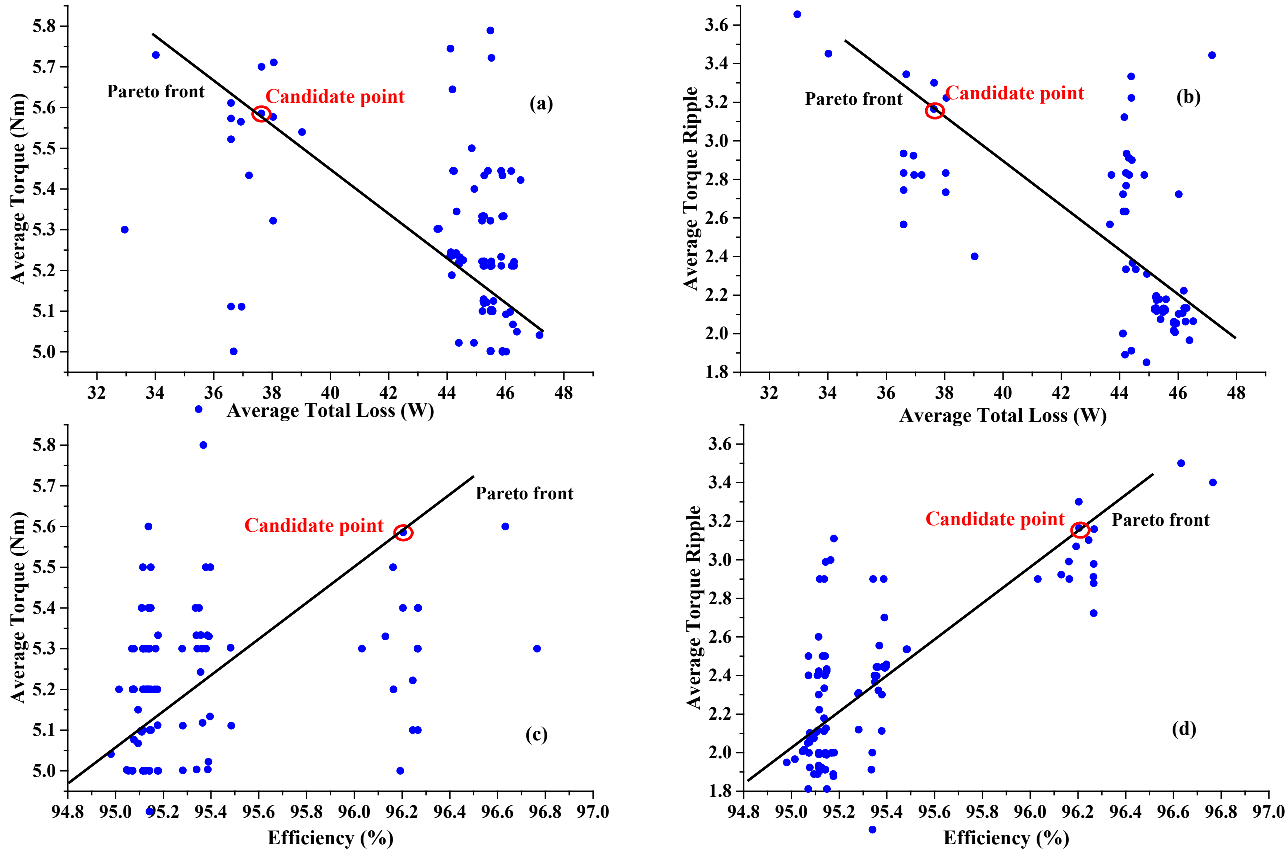

Fig. 5. Tradeoff plot of the optimization results.
(a) Torque/Loss (b) Torque Rip./Loss (c) Torque /Eff (d) Torque Rip./Eff

A. *Design of Experiment (DOE)*

Table II shows design points/solutions of DOE of the model which will be used to build a response surface. The method of DOE adopted in this work is central composite design approach. It determines how many and which design points should be solved for the most efficient approach to optimization [22].

TABLE III. DESIGN SOLUTIONS OF DOE

S/N	P1	P2	P3	P4	P5	P6	P7
1	0.38	11.65	0.38	3.964	9.11	98.23	37.86
2	0.25	11.65	0.38	4.737	12.03	98.61	35.37
3	0.50	11.65	0.38	2.683	5.56	97.28	39.92
4	0.38	10.50	0.38	3.904	9.01	98.11	39.92
5	0.38	12.80	0.38	4.027	9.24	98.32	36.62
6	0.38	11.65	0.25	3.642	9.22	94.61	110.26
7	0.38	11.65	0.50	2.238	4.51	96.54	42.58
8	0.27	10.72	0.27	3.475	8.05	96.16	73.70
9	0.48	10.72	0.27	3.721	9.62	98.12	37.84
10	0.27	12.59	0.27	3.636	8.33	96.29	74.36
11	0.48	12.59	0.27	3.856	9.83	97.16	59.88
12	0.27	10.72	0.48	3.777	8.03	80.17	496.43
13	0.48	10.72	0.48	1.499	2.75	94.07	50.17
14	0.27	12.59	0.48	3.575	8.21	94.21	116.74
15	0.48	12.59	0.48	1.529	2.82	93.95	52.33

Table III displays the initial optimisation size and a comparison of the motor output, demonstrating that the motor's performance has significantly improved following the DOE approach with reduced optimization time from 65 mins to 10 mins. P1, P2, P3, P4, P5, P6, and P7 represent Es, Ys and Er, avg. torque, efficiency, avg. torque ripple, and total loss, respectively.

B. *Sensitivity Analysis*

As shown in Fig. 3, the global sensitivity chart shows sensitivity of each of the output parameters with respect to input variables [23]. The stator pole embrace, Es, and yoke thickness, Ys, are clearly the key parameters for achieving the optimization goals. The pole-embrace of the rotor, Er, should be constrained and be closely associated with these two strategic parameters. The output parameters of average torque, average torque ripple, and efficiency, have a positive and negative sensitivity with Ys and Es respectively, while the total loss has a positive and negative sensitivity with Es and Ys respectively. All these sensitivity patterns are consistent with general motor design experience which further highlights the practical implementation and significance of the adopted optimization tool in this work. Also, the chart may be used to determine the proportion of importance.

C. *Response Surface*

Fig. 4 is the response points that show the positive trend of the input parameters with the output parameters as given by sensitivity chart of Fig. 3. The plots show the impact that the parameters have on one another.

D. *Optimization Tradeoffs*

A trade-off plot which represents the pareto front of the design is shown in Fig. 5. The plot shows the pareto points at which an improvement in the goal of an output parameter will be achieved without sacrificing another parameter under the constraints defined in the optimization program [23]. The "candidate point" in the plots represents the best design that was chosen considering the levels of priority adopted in this work.

TABLE IV. PARAMETERS OF THE CANDIDATE

Input Parameters	Values	Output Parameters	Values
Es	0.375	Tavg.	5.59 Nm
Ys	11.65 mm	T.R	3.18
Er	0.375	Eff.	96.20 %
-	-	Tloss	37.63 W

The final candidate's parameters and performance are specified in Table IV. It can be shown that the final design has the highest overall output in terms of average torque and the efficiency, with improvements of 15% and 13.2% over the base design, respectively.

V. CONCLUSION

This paper presents an optimization solution applied to a 3-phase, four pole, 1.5 kW switched reluctance motor. The finite element 2D model of the motor was analyzed using Maxwell 2D software which offers accuracy needed for the parametric simulation of the motor while the multi-objective optimization was performed in Ansys workbench

The NSGA-II intelligent optimization algorithm adopted in the work was used to optimize the parameters that have a significant impact on the objective function, reducing the optimization time from 65 mins to 10 mins and increasing the motor's optimization performance, making it a feasible and effective method of optimization. A final design that is superior to the base design was achieved based on the optimization and sensitivity analysis results.

The comprehensive performance of the optimized design in terms of average torque and efficiency were 5.59 Nm and 96.20% respectively, which are 15 % and 13.2 % better than the base model. There was a 30.1 % and 1.55 % reduction in average total loss and torque ripple in the optimized model when compared with the base design. This shows the viability of the NSGA-II intelligent optimization program as a framework to optimize the specified objective functions. However, further work is required to include more objective functions and to use this framework for specific applications.

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