Extended naked conical singularity in radiation collapse in Einstein-Gauss-Bonnet gravity

Byron P. Brassel,[*] Sunil D. Maharaj,[†] and Rituparno Goswami[‡]

Astrophysics and Cosmology Research Unit, School of Mathematics, Statistics and Computer Science, University of KwaZulu–Natal, Private Bag X54001, Durban 4000, South Africa

(Received 23 July 2018; published 11 September 2018)

In this paper, we investigate the continual gravitational collapse of a spherically symmetric radiation shell in five-dimensional Einstein-Gauss-Bonnet gravity. We show that the final fate of such a collapse is an extended and weak curvature naked conical singularity at the centre, which then subsequently becomes covered by an apparent horizon. This process is entirely different from the five dimensional general relativity counterpart, where a strong curvature singularity develops at the centre. Since the singularity in the case of Einstein-Gauss-Bonnet gravity is sufficiently weak, we argue that the spacetime can be extended through it, which gives us an elegant way of constructing regular black holes in higher dimensions without violating any energy conditions. We also extend our study to spacetimes with null and string fluids, which are the counterpart of generalized Vaidya spacetimes in general relativity. We show that similar end states are also possible in those cases. Higher-dimensional spacetimes are then considered.

DOI: 10.1103/PhysRevD.98.064013

I. INTRODUCTION

When a massive star of around 8 solar masses exhausts all of its nuclear fuel it must undergo a gravitational contraction to either a compact star or it must collapse in a continual manner without achieving an equilibrium state such as a compact stellar object. The singularity theorems of general relativity then give the prediction that the collapse must give rise to a spacetime singularity, hidden within an event horizon of gravitation or visible to the external universe. Spacetime curvature as well as energy densities become arbitrarily high at these distorted regions. When the internal dynamics of the contraction result in the delay of an event horizon formation, the singularity is theoretically visible (or naked) and may communicate effects to the external universe [1–5]. Without this delay, the covered singularity is a black hole. A singularity in any physical theory typically implies that the theory breaks down either at the vicinity of or at the singularity itself. The consequences thereof is that an alternate and more complete theory is required, in this case possibly quantum gravity, revising the given theory.

Gravitational collapse was first studied by Oppenheimer and Snyder [6] where they described the free-fall contraction of a spherical body in which the gravitational forces completely overwhelmed the outward pressure forces. With regards to dynamical and regular black holes, as well as those with trapped regions, the generalized Vaidya metric has extensively been used to study these models [7–9]. With regards to modified theories of gravity, Dominguez and Gallo [10] later studied black hole solutions in Einstein-Gauss-Bonnet gravity. The reason for considering modified theories of gravity lie in the certain shortcomings of general relativity from both the theoretical and observational points of view. Since general relativity itself is a generalisation of Newtonian gravity, extensions of general relativity are a natural notion. Introducing nonlinear forms of both the Riemann and Ricci tensor, and the Ricci scalar is one approach to modifying general relativity. Lovelock [11,12] showed that it was possible to introduce a polynomial form of the Lagrangian which is of quadratic order, and this form in turn, generated the Einstein-Gauss-Bonnet (EGB) action. Curvature terms which are quadratic within the spacetime manifold appear as corrections to Einstein gravity, and this theory can be considered as a consequence of low energy string theory [13,14].

The Boulware-Deser solution [15] was an earlier higher-dimensional analogue of the vacuum Schwarzschild metric from general relativity. A comparison between the higher-dimensional geodesic motion of a Boulware-Deser black hole and the Schwarzschild geometry was undertaken by Bhawal [16]. Some work has been done to find asymptotically AdS black hole solutions in EGB gravity [17–19]. Ghosh *et al.* [20] studied the gravitational contraction of a spherical and inhomogeneous cloud of dust in EGB gravity, while Ghosh and Maharaj [21] found null dust solutions in third order Lovelock gravity in arbitrary dimensions for a spherically symmetric string cloud background. In general

[*] drbrasselint@gmail.com
[†] maharaj@ukzn.ac.za
[‡] Goswami@ukzn.ac.za

relativity, Dawood and Ghosh [8] and Ghosh and Dawood [22] found a large family of solutions to Einstein's equations for a spherically symmetric type II fluid in four and higher dimensions, and showed that the well known black hole solutions are a particular case of this larger family. The solutions of [15] are the EGB analogues of the vacuum solutions in general relativity. An EGB Vaidya-like solution was described by Kobayashi [23] where the matter fields were analogous to those found in the four-dimensional generalized Vaidya spacetimes with type I and type II fluid distributions. Several radiating Boulware-Deser solutions were found by Brassel *et al.* [24] which were themselves the EGB analogues of those found in [25–27].

A. Regular black holes and conical singularities

The theorems of singularity formation given by Penrose and Hawking in [28] indicate the existence of singularities under certain circumstances. However it has been claimed that singularities may be unphysical objects created by the classical theories of gravity, and may not exist in the universe. These singularities were thought to be the consequence of the strong symmetry imposed on the spacetimes when deriving solutions of the Einstein field questions. An example of this would be that the singularity at $\mathsf{r} = 2m$ in the Schwarzschild metric was a coordinate singularity which could be removed via a suitable coordinate transformation. No such transformation, however, could remove the genuine curvature singularity at $\mathsf{r} = 0$. Further, it was shown that any given spacetime will admit, provided that the energy conditions and causality are not violated, singularities within some general framework [28]. Hence, under these conditions in relativity theory, singularities are a general feature. Despite this, Bardeen [29] found the first static and spherically symmetric regular black hole, which was a solution to the Einstein field equations coupled to nonlinear electrodynamics with a magnetic dipole. The Bardeen black hole was a modification of the Reisnner-Nordstöm solution. Several other regular black hole models were proposed by [30–33]. Ghosh and Amir [34] studied the horizon structure and ergosphere of a rotating Bardeen black hole and Bambi and Modesto [35] investigated rotating Bardeen and Hayward black holes by using the Newman-Janis algorithm. It must be emphasized that these prior works are contrived in the sense that coordinate transformations were used to design the regularity of the solutions, or the singularities may have been removed altogether and replaced by regular centers. Borde [30] showed that in a large class of spacetimes where the weak energy condition is satisfied, the existence of a regular black hole requires a change in the topology of the spacetimes. Further examples of these kinds of constructions can be found in the works of Frolov *et al* [36,37] and Barrabès and Frolov [38]. They discussed spacetimes with properties similar to those given above. In their cases, singularities could possibly be avoided by requiring that part of the region within the Schwarzschild radius $\mathsf{r} = 2m$, be joined to de Sitter space through a thin boundary layer.

It should be noted that all of these regular models above have similar global properties in general relativity. The difference, however, between these models and the work in this paper, is that our model does not require the use of any such construction. The metric spacetime itself, in EGB gravity, is regular throughout and this is the basis of this paper. The energy conditions are satisfied and we do not need to invoke the existence of exotic matter in our work. This is not the case in other treatments of regular black holes. However, an addition, we will show that there exists an extended weak conical singularity at the centre of the regular spacetime after the cessation of collapse. Singularities, including conical or "quasiregular" singularities were discussed in Ellis and Schmidt [39,40]. In the latter paper, various theorems were proved regarding the behavior, existence and stability of these singularities. Tod [41] obtained, by identification of flat space, several metric spacetimes with different kinds of conical singularities. Oliveira-Neto [42] used a method relating to the holonomy of a spacetime to identify conical singularity existence.

B. This paper

In this paper we investigate the continual gravitational collapse of a spherically symmetric radiation shell in five-dimensional EGB gravity, to explicitly show that the final fate of such a process is an extended and weak naked conical singularity at the center, which then subsequently gets covered by an apparent horizon. We compare this with the scenario of five-dimensional general relativity where a necessarily strong curvature singularity develops at the center. We also argue that since this conical singularity is weak enough to be extendable. This is an elegant way of naturally constructing a regular black hole in higher dimensional spacetimes. The paper is organized as follows: In the next section we give a brief overview of the already known results of five-dimensional null shell collapse in general relativity. In Sec. III, we investigate the collapse of a similar form of matter in EGB gravity, and explicitly highlight the basic differences in the nature of the singularities and boundaries of trapped regions. In Sec. IV, we then generalize the analysis to collapse with an additional string fluid, and following this, we consider higher-dimensional spacetimes.

II. RADIATION SHELL COLLAPSE IN FIVE-DIMENSIONAL GENERAL RELATIVITY: A BRIEF RECAP

In five dimensions, the collapsing pure Vaidya spacetime reads as

$$ds^2 = -\left(1 - \frac{m(v)}{\mathsf{r}^2}\right)dv^2 + 2dvd\mathsf{r} + \mathsf{r}^2(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2), \quad (1)$$

where $m(v)$ is the gravitational mass of the body. The energy momentum tensor for this kind of matter is given by

$$T_{ab} = \mu l_a l_b, \tag{2}$$

where $l_a = \delta_a^0$. The null vector l^a is a double null eigenvector of the energy momentum tensor (2). Therefore, the only field equation $G^a{}_b = \kappa T^a{}_b$ is given by

$$\mu = \frac{3}{\kappa \mathsf{r}^3} m_v, \tag{3}$$

where the subscript denotes differentiation with respect to the time coordinate v. We note that for the weak energy condition to be satisfied we must have

$$\frac{\partial m(v)}{\partial v} \geq 0. \tag{4}$$

Let us now consider the continual collapse of a five-dimensional radiation shell described by $0 \leq v \leq V_0$ (see Fig. 1). For a proper spacetime matching, in the interior of the shell we must have $m(0) = 0$, and that makes the spacetime region $v < 0$ that of Minkowski. The exterior of the shell ($v > V_0$) is matched naturally with a five-dimensional Schwarzschild spacetime with the Schwarzschild mass $M = m(V_0)$. The spacetime singularity is then located at ($v \geq 0$, $\mathsf{r} = 0$). For the Vaidya spacetime in $5-D$ the Kretschmann scalar is given by

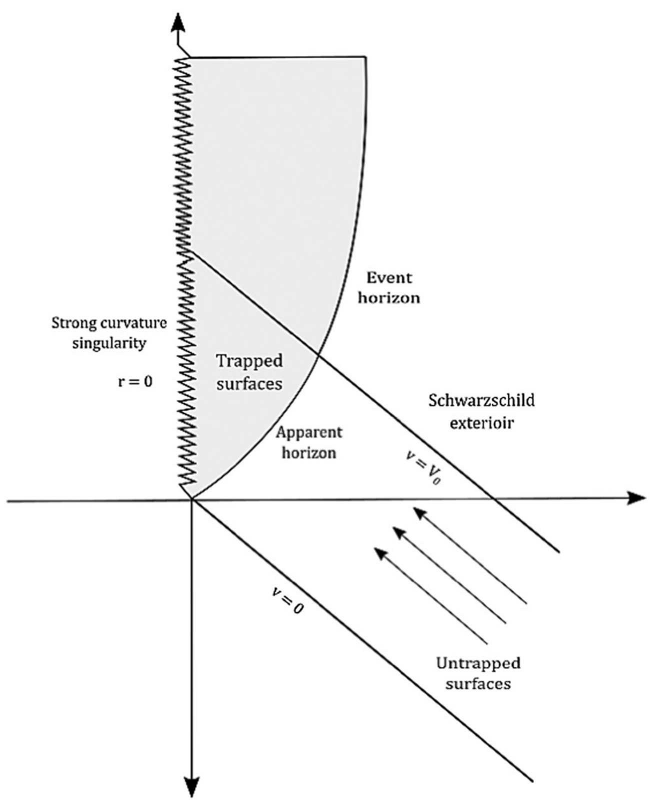

FIG. 1. Spacetime diagram of null shell collapse in $5-D$ general relativity.

$$K = \frac{72 m(v)^2}{\mathsf{r}^8}. \tag{5}$$

It can clearly be seen that at $\mathsf{r} = 0$, the above invariant diverges as $K \approx \mathsf{r}^{-8}$, hence there exists a strong curvature singularity at the center.

The boundary of the trapped surface (or the apparent horizon) is given by

$$\left(1 - \frac{m(v)}{\mathsf{r}^2}\right) = 0, \tag{6}$$

which is to say $\mathsf{r} = \sqrt{m(v)}$. The apparent horizon starts at the initial singular point ($v = 0$, $\mathsf{r} = 0$), and extends outwards into the future, where it then matches smoothly to the event horizon of exterior Schwarzschild spacetime. Singularities for the Vaidya spacetime have been extensively studied by [1–5], and it has been shown that there exists an open set of parameter spaces for the mass function $m(v)$, for which the initial singular point ($v = 0$, $\mathsf{r} = 0$) can be locally naked. However the singular points ($v > 0$, $\mathsf{r} = 0$) are covered by the horizon. It is important to note that geodesics can cross the last collapsing thin shell before the formation of the apparent horizon and so globally naked singularities are possible.

III. RADIATION SHELL COLLAPSE IN FIVE-DIMENSIONAL EGB GRAVITY

We will now investigate the same process of radiation shell collapse with the same form of null matter described by the energy momentum tensor (2), but in the case of five dimensional EGB gravity. In this theory, the modified form of the Einstein-Hilbert action in five dimensions is

$$S = -\frac{1}{16\pi} \int \sqrt{-g}[(R - 2\Lambda + \alpha L_{GB})] d^5x + S_{\text{matter}}. \tag{7}$$

The above action is called the Gauss-Bonnet action where α is the EGB coupling constant, R is the five-dimensional Ricci scalar, L_{GB} is the Lovelock term and Λ is the cosmological constant. The above action has no direct usefulness in dimensions of four or less since the Lovelock term does not contribute to the field equations, but is in general, nonvanishing in dimensions higher than four. The importance of the Lovelock term lies in the fact that the equations of motion are second order and quasilinear despite the fact that the Langrangian is quadratic in the curvature tensors and the Ricci scalar.

A. The EGB field equations

Varying the above action with respect to the metric we get the required EGB field equations:

$$\mathcal{G}_{ab} = \kappa T_{ab}, \tag{8}$$

where

$$\mathcal{G}_{ab} = G_{ab} - \frac{\alpha}{2} H_{ab}. \tag{9}$$

In the above, G_{ab} is the Einstein tensor, T_{ab} is the energy momentum tensor and H_{ab} is the Lanczos tensor which is defined as

$$H_{ab} = g_{ab} L_{\rm GB} - 4 R R_{ab} + 8 R_{ac} R^{c}{}_{b} + 8 R_{acbd} R^{cd} - 4 R_{acde} R_{b}{}^{cde}, \tag{10}$$

where the Lovelock term is given by

$$L_{\rm GB} = R^2 + R_{abcd} R^{abcd} - 4 R_{cd} R^{cd}. \tag{11}$$

In the limit where $\alpha \to 0$, the Lanczos term vanishes and Einstein gravity will be regained.

B. Solution to the field equations for spherically symmetric null fluid

Let us consider the energy momentum tensor of the null fluid to be the same as general relativity,

$$T_{ab} = \tilde{\mu} l_a l_b, \tag{12}$$

where $l_a = \delta^0_a$. Also let us consider the metric ansatz in the following form:

$$ds^2 = -f(v, \mathsf{r}) dv^2 + 2 dv d\mathsf{r} + \mathsf{r}^2 (d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2). \tag{13}$$

Solving the EGB field equations for the energy momentum tensor (12), gives the following solution

$$f(v, \mathsf{r}) = 1 + \frac{\mathsf{r}^2}{4\alpha} \left(1 - \sqrt{1 + \frac{16 M(v) \alpha}{\mathsf{r}^4}} \right), \tag{14}$$

where the function $M(v)$ is a solution of the EGB field equation

$$\tilde{\mu} = \frac{3}{\kappa \mathsf{r}^3} M_v. \tag{15}$$

Again we note that just like general relativity, for the weak energy condition to be satisfied we must have

$$\frac{\partial M(v)}{\partial v} \geq 0. \tag{16}$$

C. Collapsing radiation shell, singularity and apparent horizon

Again, just as in GR, consider the continual collapse of a five dimensional radiation shell described by $0 \leq v \leq V_0$. It is easy to see from the solution (14) that $M(0) = 0$ makes the spacetime region for $v < 0$ flat spacetime, whereas the exterior of the shell ($v > V_0$) is matched naturally with a five-dimensional Boulware-Deser spacetime with the mass $M_0 = M(V_0)$. The metric of the exterior spacetime is then given as

$$ds^2 = -f(\mathsf{r}) dv^2 + 2 dv d\mathsf{r} + \mathsf{r}^2 (d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2), \tag{17}$$

where

$$f(\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{4\alpha} \left(1 - \sqrt{1 + \frac{16 M_0 \alpha}{\mathsf{r}^4}} \right). \tag{18}$$

Now, interestingly, a closer inspection of the metric (14) reveals that all the metric functions are well defined and regular at $v \geq 0$, $\mathsf{r} = 0$, depicting the absence of a real strong curvature spacetime singularity. To understand the above observation more carefully, we calculate the Kretschmann invariant, which is given by

$$K = -\frac{1}{2\alpha \mathsf{r}^4} (\mathsf{r}^2 + 16\alpha M)^{-3} \Big[\left(-\mathsf{r}^2 + \sqrt{\mathsf{r}^4 + 16\alpha M} \right)^2 \times \Big(-7\mathsf{r}^{12} - 2\mathsf{r}^{10} \sqrt{\mathsf{r}^4 + 16\alpha M} - 184 \alpha \mathsf{r}^8 M - 2048 \alpha^2 \mathsf{r}^4 M^2 - 6144 \alpha^3 M^3 + 32 \alpha \mathsf{r}^6 M \sqrt{\mathsf{r}^4 + 16\alpha M} \Big) \Big]. \tag{19}$$

We can immediately see that the Kretschmann invariant diverges as $K \approx r^{-4}$, which is a much slower divergence than the GR case. Figure 2 shows the behavior of various curvature scalars, where we have chosen $\alpha = 2$ and $M(v) = v$. The slower divergence denotes a singularity which is weak in nature, and furthermore the metric functions being well defined at $v \geq 0$, $\mathsf{r} = 0$ depicts the emergence of a weak conical singularity at the center.

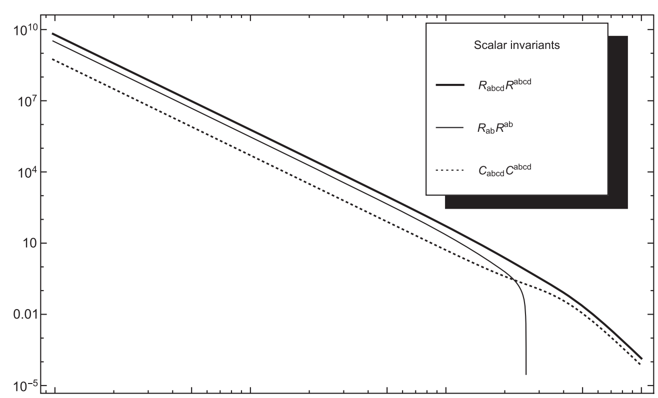

FIG. 2. Log-log plot showing the behavior of the Kretschmann scalar, the Ricci tensor squared and the Weyl tensor squared.

To see the dynamics of the apparent horizon, which is the boundary of the trapped region, we consider

$$f(v,\mathsf{r}) = 0, \tag{20}$$

which is to say that

$$1 + \frac{\mathsf{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16\alpha M(v)}{\mathsf{r}^4}}\right) = 0. \tag{21}$$

Solving for r we get

$$\mathsf{r} = \sqrt{2(M(v) - \alpha)}. \tag{22}$$

This implies that for $v \in [0, M^{-1}(\alpha)]$, the conical singularity at the center ($\mathsf{r} = 0$) remains naked, before it eventually becomes trapped.

Now since the singularity at the center is a sufficiently weak conical singularity, where the metric functions remain regular, it is evident that this singularity can be resolved, that is, one can in principle construct a spacetime extension through it. This gives us an elegant way to construct a regular black hole in higher dimensions without violating any energy conditions. To illustrate this more transparently let us assume, e.g., the case when

$$M(v) = \lambda v. \tag{23}$$

Equation (22) becomes

$$\mathsf{r} = \sqrt{2\lambda v - 2\alpha}. \tag{24}$$

At $\mathsf{r} = 0$, we have that $v \neq 0$ (as would be the case in Einstein gravity) but is in fact $v = \alpha/\lambda$. So this conical singularity has a window period of existence in which it is uncovered, and this depends solely on the Gauss-Bonnet term α, in the sense that it delays the horizon formation. This is the fundamental difference between EGB gravity and general relativity. In the case of $\alpha = 0$ the apparent horizon would form at $\mathsf{r} = v = 0$ covering the singularity. We can describe the nature of the apparent horizon and the trajectories of null geodesics graphically. In Figs. 3 and 4 a radiating matter distribution is focused into a five-dimensional regular black hole. In the region $0 < v < \frac{\alpha}{\lambda}$ there is a conical singularity with no trapping horizon as the Gauss-Bonnet term α delays the formation of the apparent horizon in this region. The apparent horizon forms at $v = \frac{\alpha}{\lambda}$ and encloses a compact region of trapped surfaces for $\frac{\alpha}{\lambda} < v < V_0$. At $v = V_0$ the apparent and event horizons join smoothly as a single trapping event horizon at $\mathsf{r} = \sqrt{2\lambda v - 2\alpha}$ separating the exterior Boulware-Deser vacuum from the trapped surfaces. Beyond the event horizon is a black hole with a quasiregular (or weak conically singular) center.

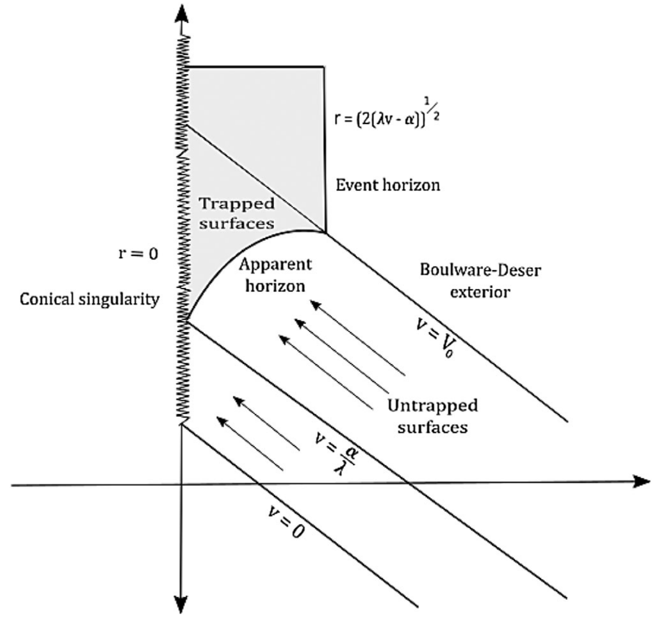

FIG. 3. Spacetime diagram depicting matter falling into a black hole with conical singularity. The nonzero Gauss-Bonnet constant α acts as a delay term in the formation of the apparent horizon.

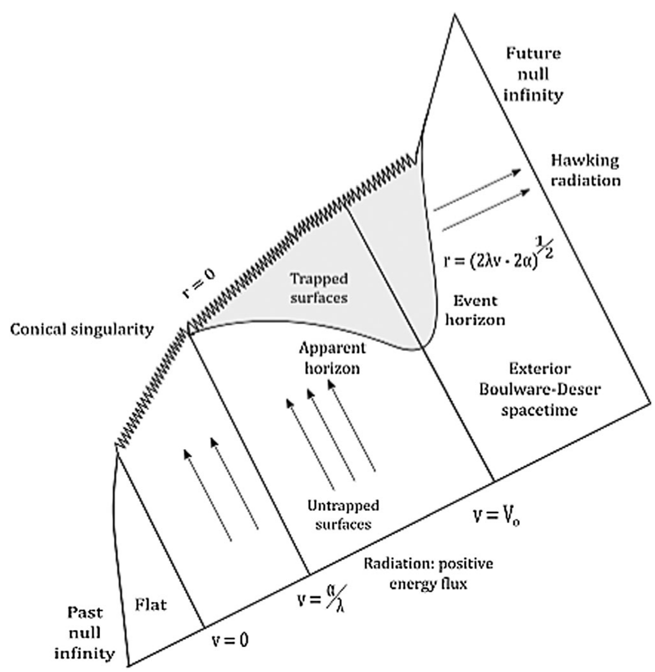

FIG. 4. Penrose diagram with a depiction of matter falling into a black hole. There is an extended weak conical singularity formation in the region $0 < v < \frac{\alpha}{\lambda}$ and there exist families of trajectories escaping to infinity from the black hole. The apparent horizon only begins to form in the region $\frac{\alpha}{\lambda} < v < V_0$. We have a radiating distribution of matter focused into the regular black hole region and at $v = \frac{\alpha}{\lambda}$, a shell of null radiation falls through this radiating distribution of matter and into the black hole.

TABLE I. Comparison between Vaidya and Boulware-Deser spacetimes.

Spacetime	Vaidya	Boulware-Deser
Theory of gravity	Einstein	Einstein-Gauss-Bonnet
Regularity	No	Yes
Dimensions (N)	All $N \geq 4$	$5 = N \neq 6, 7, \ldots$
Kretschmann scalar	Divergent	Divergent
Other diffeomorphism invariants	All divergent	All divergent
Singularity existence	Yes	Yes
Singularity type	Curvature	Conical
Singularity strength	Strong	Weak

We finally present a comparison in Table I between the Vaidya spacetime in general relativity and the radiating Boulware-Deser spacetime in EGB gravity.

IV. EXTENSION OF OUR RESULTS TO GENERALIZED VAIDYA-LIKE SPACETIMES IN EGB GRAVITY

An inhomogeneous and radiating spacetime in EGB gravity is possible if we allow the mass function to depend on both the radius of the star r and the retarded null coordinate v

$$\tilde{M} \to M(v, \mathsf{r}). \tag{25}$$

We will then have

$$ds^2 = -f(v, \mathsf{r})dv^2 + 2dvd\mathsf{r} + \mathsf{r}^2(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2), \tag{26}$$

with

$$f(v, \mathsf{r}) = 1 + \frac{\mathsf{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16M(v, \mathsf{r})\alpha}{\mathsf{r}^4}}\right). \tag{27}$$

For generalized matter with a string fluid an energy momentum tensor is given by

$$T_{ab} = \tilde{\mu} l_a l_b + (\tilde{\rho} + P)(l_a n_b + l_b n_a) + P g_{ab}, \tag{28}$$

where we have that

$$l_a = \delta_a^0,$$
$$n_a = \frac{1}{2}\left[1 + \frac{\mathsf{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16M\alpha}{\mathsf{r}^4}}\right)\right]\delta_a^0 + \delta_a^1,$$

with $l_c l^c = n_c n^c = 0$ and $l_c n^c = -1$. The null vector l^a is a double null eigenvector of the energy momentum tensor (28). The EGB field equations $\mathcal{G}^a{}_b = \kappa T^a{}_b$ hence become

$$\tilde{\mu} = \frac{3}{\kappa \mathsf{r}^3} M_v, \tag{29a}$$

$$\tilde{\rho} = \frac{3}{\kappa \mathsf{r}^3} M_{\mathsf{r}}, \tag{29b}$$

$$P = -\frac{1}{\kappa \mathsf{r}^2} M_{\mathsf{rr}}. \tag{29c}$$

When $\tilde{\rho} = P = 0$, the above equations reduce to the single solution (15) obtained for the case of the radiating Vaidya-Boulware-Deser metric. Furthermore when $\tilde{\mu} = \tilde{\rho} = P = 0$, we reacquire the original vacuum case with constant mass. For this kind of fluid, the energy conditions become

(1) The weak and strong energy conditions:

$$\tilde{\mu} \geq 0, \qquad \tilde{\rho} \geq 0, \qquad P \geq 0 \qquad (\tilde{\mu} \neq 0). \tag{30}$$

(2) The dominant energy condition:

$$\tilde{\mu} \geq 0, \qquad \tilde{\rho} \geq P \geq 0 \qquad (\tilde{\mu} \neq 0). \tag{31}$$

When the mass function reduces to $M = M(v)$ the above energy conditions all reduce to $\tilde{\mu} \geq 0$ as presented earlier, and if $M = M(\mathsf{r})$, then we have that $\tilde{\mu} = 0$ and the matter field becomes a type I fluid.

Now, from the solution (27) it is clear that whenever the mass function $M(v, \mathsf{r})$, is regular and least C^2 at the centre $\mathsf{r} = 0$, we get a similar situation as in the previous section. In other words, there exist open sets of possible mass functions $M(v, \mathsf{r})$, for which the continual collapse ends in the formation of a naked and extended weak conical singularity, that in principle can be resolved with an extension of the spacetime manifold. The trapping horizon formation is delayed by the Gauss-Bonnet constant α, however it eventually forms, covering the conical singularity. Hence, we can now make the following statement:

Consider a five-dimensional collapsing inhomogeneous and radiating Boulware-Deser spacetime from a regular epoch, with a generalized mass function $M(v, \mathsf{r})$ that obeys all physically reasonable energy conditions and is at least twice differentiable in the entire spacetime. Such a spacetime is always regular. Although the Kretschmann invariant is positive and divergent at the center, the divergence is sufficiently weak and so the collapse will always terminate with the formation of an extended naked weak curvature conical singularity, and eventually a black hole with a conical singularity. Resolving this weak conical singularity will make this a dynamically formed regular black hole with no violation of the energy conditions.

V. HIGHER-DIMENSIONAL BOULWARE-DESER SPACETIMES

The gravitational collapse scenario of the Boulware-Deser spacetime differs in higher dimensions. In fact, in dimensions $N > 5$, the collapse will always terminate with a singularity, which may or may not be naked. The N-dimensional purely radiating Boulware-Deser metric is given by

$$ds^2 = -f(v,\mathsf{r})dv^2 + 2dvd\mathsf{r} + \mathsf{r}^2 d\Omega_{N-2}^2, \tag{32}$$

with

$$d\Omega_{N-2}^2 = \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](d\theta^i)^2,$$

and where

$$f(v,\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M(v)}{\mathsf{r}^{N-1}}\right)}\right).$$

In the above we have that $\hat{\alpha} = \alpha(N-3)(N-4)$.

The above metric is singular for all $N > 5$. The formation of the apparent horizon occurs when

$$f(v,\mathsf{r}) = 0, \tag{33}$$

which is to say

$$1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M(v)}{\mathsf{r}^{N-1}}\right)}\right) = 0. \tag{34}$$

It is important to note that if $N > 5$, $f(v,\mathsf{r})$ tends to infinity. The above equation cannot be solved explicitly for r, however, we can write it as

$$\frac{4M(v)}{N-3} = \mathsf{r}^{N-5}(\hat{\alpha} + \mathsf{r}^2). \tag{35}$$

An explicit solution is possible if the dimension N is specified. If we assume, e.g., that $N = 6$, then Eq. (35) becomes quadratic

$$\mathsf{r}^2 + \hat{\alpha}\mathsf{r} - \frac{4M}{3} = 0, \tag{36}$$

and admits the solutions

$$\mathsf{r} = -\frac{1}{2}\hat{\alpha} \pm \frac{1}{6}\sqrt{9\hat{\alpha}^2 + 48M(v)}. \tag{37}$$

Following the same ansatz as earlier, we let $M(v) = \lambda v$ where λ is a constant. We then have

$$\mathsf{r} = -\frac{1}{2}\hat{\alpha} \pm \frac{1}{6}\sqrt{9\hat{\alpha}^2 + 48\lambda v}. \tag{38}$$

At $\mathsf{r} = 0$ we have that $v = 0$ unlike in the five-dimensional case, so the apparent horizon forms at $\mathsf{r} = v = 0$. In the case of $\alpha = 0$, the apparent horizon also forms at $\mathsf{r} = v = 0$ as is the case in Einstein gravity. Therefore in dimensions six and higher, the Boulware-Deser spacetime is singular and the Gauss-Bonnet constant α has no affect on the collapse dynamics; there is no delay in the formation of the horizon caused by the α term.

VI. CONCLUSION

In this paper we transparently demonstrated that the final outcome of the continual gravitational collapse of a spherically symmetric radiation shell in five-dimensional EGB gravity is an extended and weak naked conical singularity at the centre, which then subsequently becomes covered by an apparent horizon. The trapping horizon formation is delayed by the presence of the Gauss-Bonnet constant α. This scenario is dramatically different from five-dimensional general relativity, where during the collapse of a radiation shell, a necessarily strong curvature singularity develops at the centre. We then extended this result to generalized Vaidya-like spacetimes to show that whenever the mass function remains regular and at least twice differentiable in the entire spacetime, an absolutely similar picture emerges.

Now, since the conical singularity that develops at the centre is weak enough to be extendable, this is an elegant way of naturally constructing a regular black hole in higher dimensional spacetimes, without violating any energy conditions.

ACKNOWLEDGMENTS

B. P. B., S. D. M. and R. G. thank the University of KwaZulu–Natal for its continued support. S. D. M. acknowledges that this work is based upon research supported by the South African Research Chair Initiative of the Department of Science and Technology and the National Research Foundation.

[1] P. S. Joshi, *Global Aspects in Gravitation and Cosmology* (Clarendon Press, Oxford, 1993).
[2] P. S. Joshi, *Gravitational Collapse and Spacetime Singularities* (Cambridge University Press, Cambridge, England, 2007).
[3] M. D. Mkenyeleye, R. Goswami, and S. D. Maharaj, Phys. Rev. D **90**, 064034 (2014).
[4] M. D. Mkenyeleye, R. Goswami, and S. D. Maharaj, Phys. Rev. D **92**, 024041 (2015).
[5] B. P. Brassel, R. Goswami, and S. D. Maharaj, Phys. Rev. D **95**, 124051 (2017).
[6] J. R. Oppenheimer and H. Snyder, Phys. Rev. **56**, 455 (1939).
[7] S. A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006).
[8] A. K. Dawood and S. G. Ghosh, Phys. Rev. D **70**, 104010 (2004).
[9] V. P. Frolov, J. High Energy Phys. 05 (2014) 049.
[10] A. E. Dominguez and E. Gallo, Phys. Rev. D **73**, 064018 (2006).
[11] D. Lovelock, J. Math. Phys. (N.Y.) **12**, 498 (1971).
[12] D. Lovelock, J. Math. Phys. (N.Y.) **13**, 874 (1972).
[13] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **163B**, 123 (1985).
[14] R. R. Metsaev, M. A. Rahmanov, and A. A. Tseytlin, Phys. Lett. B **193**, 207 (1987).
[15] D. G. Boulware and S. Deser, Phys. Lett. **55**, 2656 (1985).
[16] B. Bhawal, Phys. Rev. D **42**, 449 (1990).
[17] J. T. Wheeler, Nucl. Phys. **B268**, 737 (1986).
[18] D. L. Wiltshire, Phys. Lett. **169B**, 36 (1986).
[19] R. G. Cai, Phys. Rev. D **65**, 084014 (2002).
[20] S. G. Ghosh, S. Jhingan, and D. W. Deshkar, J. Phys. Conf. Ser. **484**, 012013 (2014).
[21] S. G. Ghosh and S. D. Maharaj, Phys. Rev. D **89**, 084027 (2014).
[22] S. G. Ghosh and A. K. Dawood, Gen. Relativ. Gravit. **40**, 9 (2008).
[23] T. Kobayashi, Gen. Relativ. Gravit. **37**, 1869 (2005).
[24] B. P. Brassel, S. D. Maharaj, and R. Goswami, Gen. Relativ. Gravit. **49**, 101 (2017).
[25] V. Husain, Phys. Rev. D **53**, R1759 (1996).
[26] A. Wang and Y. Wu, Gen. Relativ. Gravit. **31**, 107 (1999).
[27] B. P. Brassel, S. D. Maharaj, and R. Goswami, Gen. Relativ. Gravit. **49**, 37 (2017).
[28] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge University Press, Cambridge, England, 1973).
[29] J. M. Bardeen, *Abstracts of the 5th International Conference on Gravitation and the Theory of Relativity*, edited by V. A. Fock *et al.* (Tbilisi University Press, Tbilisi, 1968), p. 174.
[30] A. Borde, Phys. Rev. D **50**, 3692 (1994).
[31] S. A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006).
[32] S. G. Ghosh and S. D. Maharaj, Eur. Phys. J. C **75**, 7 (2015).
[33] I. Dymnikova and E. Galaktionov, Classical Quantum Gravity **32**, 165015 (2015).
[34] S. G. Ghosh and M. Amir, Eur. Phys. J. C **75**, 553 (2015).
[35] C. Bambi and L. Modesto, Phys. Lett. B **721**, 329 (2013).
[36] V. P. Frolov, M. A. Markov, and V. F. Mukhanov, Phys. Lett. B **216**, 272 (1989).
[37] V. P. Frolov, M. A. Markov, and V. F. Mukhanov, Phys. Rev. D **41**, 383 (1990).
[38] C. Barrabès and V.P Frolov, Phys. Rev. D **53**, 3215 (1996).
[39] G. F. R. Ellis and B. G. Schmidt, Gen. Relativ. Gravit. **8**, 915 (1977).
[40] G. F. R. Ellis and B. G. Schmidt, Gen. Relativ. Gravit. **10**, 989 (1979).
[41] K. P. Tod, Classical Quantum Gravity **11**, 1331 (1994).
[42] G. Oliveira-Nero, J. Math. Phys. (N.Y.) **37**, 4716 (1996).