Higher-dimensional radiating black holes in Einstein-Gauss-Bonnet gravity

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The higher order curvature corrections in Einstein-Gauss-Bonnet gravity play a significant role in the dynamics of gravitational collapse. We extend the gravitational collapse of radiating shells of matter in Einstein-Gauss-Bonnet gravity to higher dimensions, in the context of the cosmic censorship conjecture. In five dimensions the final collapse terminates with the formation of an extended and weak conical, naked singularity in the central region. For dimensions $N > 5$, we determine that collapse terminates with a strong curvature singularity which may or may not be naked. Cosmic censorship is affected by higher-order curvature corrections. A comparison with the higher-dimensional general relativity counterpart is also given, where the dynamics are affected by the higher dimensions.

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I. INTRODUCTION

When a massive star of mass greater than $8M_\odot$ (but less than $40M_\odot$) reaches the end of the luminous phase of its life, it experiences a gravitational collapse which is inwardly directed. This violent process is observed as a type II supernova and lasts several seconds, due to the very quick duration (a few days) of the silicon burning process, the final burning stage of the progenitor star. For truly massive stars, of the order of $40M_\odot$ to $50M_\odot$ and beyond, the collapse is observed as a superluminous supernova (SLSN) (or hypernova). An immense amount of energy (radiation, convection and conduction) is ejected from the star in the form of neutrinos or photons and so the radiation effects are important in these later stages [1]. In this regard, gravitational collapse remains a truly important endeavor in the research areas of nuclear physics, stellar astrophysics, high energy physics and general relativity.

Once the hydrogen has burned out in the star's core, the next phase of thermonuclear burning—helium—commences. Hydrogen in some surrounding shell will continue to burn. Further concentric burning shells are created as one element after the other is synthesized. Neutrino pairs are created by the annihilation of electron-positron pairs in the core. At the exhaustion of each elemental fuel, the core contracts further until the ignition temperature for the next step in the next chain is attained, and it is these reignition steps that halt the collapse. Each successive burning stage is quicker than the preceding one, however each releases less energy than the previous since the atomic nuclei become progressively heavier. ^{56}Fe is the end point of nucleosynthesis (the result of the above mentioned silicon burning). A hydrodynamical instability sets in, where the inward pressure of gravity in the star begins to overwhelm the outward energy being released. A catastrophic collapse then supervenes where gravity crushes the core to such an extent that electrons become relativistic and the resulting collapsed core remnant, and the supernova remnant, are pushed away from each other during this phase. The core remnant is either a more compact object like a neutron star (which may itself collapse further at a later time), or in the case of truly massive stars, a stellar-mass black hole with a central singularity which may or may not be naked. Incidentally, the supernova remnant expands outward at very high speed (of the order of 10^4 km/s) interacting with interstellar gases, and is in fact visible at all wavelengths between the radio and x-rays.

Within a wide variety of gravitational theories, the theorems of singularity formation foretell the phenomenon of spacetime singularities as the end states of gravitational collapse [2]. Their occurrence depends upon some generic conditions such as the existence of trapped surfaces, causality not being violated and the general attractive nature of gravitation itself. The singularity theorems also show that there exist a large class of solutions to Einstein's field equations which are geodesically incomplete. The theorems, however, do not regard the nature of the spacetime singularities themselves, i.e., whether it is possible for future directed null geodesics to escape to infinity from the close vicinity of the singularities, leaving them naked. The cosmic censorship conjecture was proposed by Penrose [3] to avoid such situations; a physically reasonable matter field which collapses under its own gravity must result in the formation of a spacetime singularity which is covered at

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all times by a trapping horizon. Therefore, the end state of continual gravitational contraction must be a black hole, with a central spacetime curvature singularity, covered by an event horizon shielding it from any and all external observers.

A general proof of the conjecture remains evasive as it stands, and several counterexamples for particular matter distributions exist in the literature. Spherically symmetric and dynamical gravitational collapse in general relativity has been extensively studied and various models exist which indicate that, based on initial data, collapse terminates in the formation of locally naked singularities [4–10]. The trapped surfaces do not form in a timely enough manner to shield the singularity from the external universe in the above cases. These studies also show that families of outgoing future nonspacelike geodesics emanate from the naked singularity, escaping to infinity [7,11]. The one drawback of the above models is that censorship violation depends on symmetries of spacetime.

It was shown in [9,12,13] that naked singularities arising as a result of dust collapse, in general relativity, were eliminated when the transition to higher dimensions was made; cosmic censorship was restored in higher dimensions under specific physical conditions like the smoothness of the initial data from which the collapse develops in time. A key aspect of this paper, is that we will demonstrate that for null radiating matter in higher-dimensional modified gravity, it will always hold; cosmic censorship is never violated, and this is irrespective of the increase in dimensions. Dimensions play no part in the end state of collapse.

In modified theories of gravity, Dominguez and Gallo [14] studied black hole solutions in Einstein-Gauss-Bonnet (EGB) gravity while Ghosh *et al.* [15] discussed the inhomogeneous gravitational collapse of a spherical dust cloud in the same theory. Some asymptotically AdS black hole solutions have also been found by [16–18] in EGB gravity. Brassel *et al.* [19] recently studied the collapse of a radiating shell in five-dimensional EGB gravity, and found that the spacetime itself was inherently regular and the collapse terminated with the formation of a black hole with an extended and weak conical singularity. This is fundamentally different to the five-dimensional general relativity counterpart. A brief recap of this is given in a later section.

A. This paper

In this paper, we will extend on the above notions and consider higher-dimensional collapse in EGB gravity. We will demonstrate that the dynamics of collapse are considerably different than in the five-dimensional case. In the following section of the paper, we provide a brief recap on radiation collapse in arbitrary dimensions in the framework of general relativity. In Sec. III we discuss N-dimensional EGB gravity before giving a recap on radiating gravitational collapse in five dimensions [19], in Sec. IV. The following sections contain the main aspects of this paper. Section V deals with higher-dimensional radiation shell collapse in EGB gravity. We demonstrate that the end state of collapse is a strong central curvature singularity, which is different to the five-dimensional collapse model. A comparison is undertaken between the five-dimensional and higher-dimensional models and plots are given for various curvature invariants. Sections VI–VIII deal with the analysis of the singularity, post collapse; whether naked singularities form or not cannot be determined by the analysis, i.e., cosmic censorship may or may not be violated. Finally, a brief description and recap of collapse in higher-order Lovelock gravity is given in Sec. IX.

II. RADIATION SHELL COLLAPSE IN HIGHER-DIMENSIONAL GENERAL RELATIVITY

In N dimensions, the collapsing pure Vaidya metric is given by

$$ds^2 = -\left(1 - \frac{2m(v)}{(N-3)\mathsf{r}^{N-3}}\right)dv^2 + 2dvd\mathsf{r} + \mathsf{r}^2 d\Omega^2_{N-2}, \tag{1}$$

with

$$d\Omega^2_{N-2} = \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](d\theta^i)^2.$$

In the above, $m(v)$ is the gravitational mass of the body and $N \geq 4$. For this kind of matter, the energy momentum tensor is given by

$$T_{ab} = \mu l_a l_b, \tag{2}$$

where $l_a = \delta^0_a$. The vector l^a is in fact a double null eigenvector of the energy momentum tensor (2). The only field equation is thus

$$\mu = \frac{(N-2)m_v}{\mathsf{r}^{N-2}}, \tag{3}$$

where the subscript denotes differentiation with respect to the temporal coordinate v. In order for the weak energy condition to be satisfied, it is imperative that

$$\frac{\partial m(v)}{\partial v} \geq 0. \tag{4}$$

Figure 1 depicts the continual contraction of an N-dimensional radiation shell described by $0 \leq v \leq V_0$. With regards to a proper spacetime matching, $m(0) = 0$ in the interior of the shell which implies that the region $v < 0$ is Minkowski spacetime. The exterior of the radiation shell, $v > V_0$, is matched with an N-dimensional Schwarzschild metric with the mass $M = m(V_0)$. The singularity of the

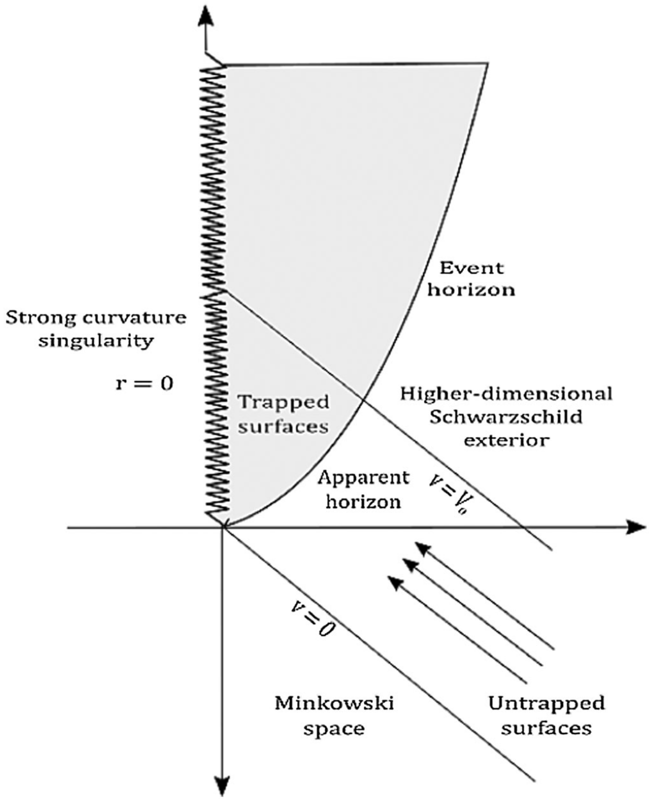

FIG. 1. Spacetime diagram of null shell collapse in N-dimensional general relativity.

spacetime is then located at ($v \geq 0$, $\mathsf{r} = 0$). The Kretschmann invariant ($K = R_{abcd}R^{abcd}$) for (1) is given by

$$K = 4\frac{(N-1)(N-2)^2}{N-3}\frac{m(v)^2}{\mathsf{r}^{2(N-1)}}, \tag{5}$$

which clearly diverges as $K \approx \mathsf{r}^{-2N+2}$ at $\mathsf{r} = 0$. Hence, there is a strong curvature singularity at the center of the fluid distribution.

The apparent horizon forms when

$$\left(1 - \frac{2m(v)}{(N-3)\mathsf{r}^{N-3}}\right) = 0, \tag{6}$$

which is to say that the boundary of the trapped surface is given by

$$\mathsf{r} = \left(\frac{2m(v)}{N-3}\right)^{\frac{1}{N-3}}. \tag{7}$$

The boundary of the trapped surface begins at the singular point ($v = 0$, $\mathsf{r} = 0$) and is extended outward into the future where it is matched to the event horizon of the Schwarzschild exterior spacetime. The singularities of the Vaidya spacetime have been studied in detail by Joshi [6,7] where it was shown that there exists a set of open parameter spaces for the mass function, for which the singular point ($v = 0$, $\mathsf{r} = 0$) can be locally and globally[1] naked. Mkenyeleye *et al.* [9] conducted a singularity analysis for spacetimes in general dimensions where it was shown that the dynamics of the collapse process are affected by the presence of higher dimensions.

III. HIGHER-DIMENSIONAL EGB GRAVITY THEORY

The purpose of this paper is to investigate the gravitational collapse of radiating spacetimes in higher dimensions in EGB gravity. The Gauss-Bonnet action in N dimensions is given by

$$S = -\frac{1}{16\pi}\int \sqrt{-g}[(R - 2\Lambda + \alpha L_{GB})]d^N x + S_{\text{matter}}, \tag{8}$$

which is a modified form of the Einstein-Hilbert action. In the above, R is the Ricci scalar, α is the EGB coupling constant and Λ is the cosmological constant.[2] Additionally, L_{GB} is the Lovelock term, given by

$$L_{GB} = R^2 + R_{abcd}R^{abcd} - 4R_{cd}R^{cd}, \tag{9}$$

which is essentially the linear combination of quadratic terms in curvature. Varying (8) with respect to the action $\delta S = 0$, yields the EGB field equations

$$\mathcal{G}_{ab} = \kappa T_{ab}, \tag{10}$$

where

$$\mathcal{G}_{ab} = G_{ab} - \frac{\alpha}{2}H_{ab}. \tag{11}$$

In the above, G_{ab} is the Einstein tensor, T_{ab} is the energy momentum tensor and H_{ab} is the Lanczos tensor which is defined as

$$H_{ab} = g_{ab}L_{GB} - 4RR_{ab} + 8R_{ac}R^{c}{}_{b} + 8R_{acbd}R^{cd} - 4R_{acde}R_{b}{}^{cde}. \tag{12}$$

In the limit where $\alpha \to 0$, the Lanczos term vanishes and Einstein gravity will be regained.

IV. RADIATION SHELL COLLAPSE IN FIVE-DIMENSIONAL EGB GRAVITY

In this section, we will provide the essential results of the work done in Brassel *et al.* [19]. Consider the following metric

[1]Global naked singularities are indeed possible because geodesics can cross the final collapsing thin shell before the formation of the apparent horizon.

[2]For the purposes of this work, Λ has been set to zero.

$$ds^2 = -f(v,\mathsf{r})dv^2 + 2dvd\mathsf{r} + \mathsf{r}^2(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2), \quad (13)$$

along with an energy momentum tensor of the null fluid,

$$T_{ab} = \tilde{\mu} l_a l_b,$$

which is similar to (2) in general relativity. Solving the EGB field equations for this energy momentum tensor, gives the following

$$f(v,\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16M(v)\alpha}{\mathsf{r}^4}}\right), \quad (14)$$

where the function $M(v)$ is itself a solution of the EGB field equation

$$\tilde{\mu} = \frac{3}{\kappa \mathsf{r}^3} M_v. \quad (15)$$

This field equation with $M(v)$ is the radiating Boulware-Deser solution. Similar to the general relativity case, we must have that

$$\frac{\partial M(v)}{\partial v} \geq 0, \quad (16)$$

for the weak energy condition to be satisfied. By observing (14), it can be seen that all of the metric functions are well defined and regular at $\mathsf{r} = 0$, $v \geq 0$, implying the absence of a strong curvature singularity. Further to this notion, the Kretschmann scalar is calculated as

$$K = -\frac{1}{2\alpha \mathsf{r}^4}(\mathsf{r}^2 + 16\alpha M)^{-3}[(-\mathsf{r}^2 + \sqrt{\mathsf{r}^4 + 16\alpha M})^2 \times (-7\mathsf{r}^{12} - 2\mathsf{r}^{10}\sqrt{\mathsf{r}^4 + 16\alpha M} - 184\alpha \mathsf{r}^8 M - 2048\alpha^2 \mathsf{r}^4 M^2 - 6144\alpha^3 M^3 + 32\alpha \mathsf{r}^6 M\sqrt{\mathsf{r}^4 + 16\alpha M})], \quad (17)$$

which diverges as $K \approx \mathsf{r}^{-4}$, a much slower divergence than the case (in any dimension) in general relativity. This slower divergence, coupled with the fact that the metric functions are all well defined at $\mathsf{r} = 0$, $v \geq 0$, is indicative of a spacetime singularity which is *conical* and *weak* in nature.

In order to gauge the dynamics of the boundary of the trapped region, we let

$$f(v,\mathsf{r}) = 0,$$

which is

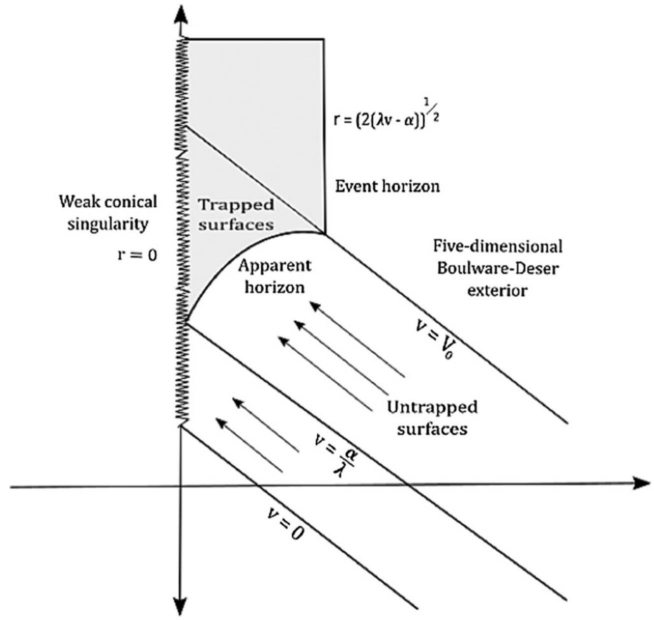

FIG. 2. Spacetime diagram of null shell collapse in five-dimensional EGB gravity.

$$1 + \frac{\mathsf{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16\alpha M(v)}{\mathsf{r}^4}}\right) = 0. \quad (18)$$

Solving for the radius r we find

$$\mathsf{r} = \sqrt{2M(v) - 2\alpha}. \quad (19)$$

The implication here is that for $v \in [0, M^{-1}(\alpha)]$, the conical singularity at the center ($\mathsf{r} = 0$) remains naked, before succumbing to the trapping horizon. If we consider Fig. 2, where $M(v) = \lambda v$, it can be observed that matter, which is radiating, is falling into a five-dimensional regular black hole. Within the region $0 < v < \frac{\alpha}{\lambda}$, there is no trapping horizon of any kind covering the conical singularity, as the Gauss-Bonnet constant α is delaying its formation. The apparent horizon finally begins to form at $v = \frac{\alpha}{\lambda}$ and encloses a region of trapped and null compact surfaces which fall into the black hole within $\frac{\alpha}{\lambda} < v < V_0$. At $v = V_0$ a single event horizon separates the exterior 5-*D* Boulware-Deser vacuum from the trapped surfaces at $\mathsf{r} = \sqrt{2\lambda v - 2\alpha}$. Beyond this final event horizon is a black hole with a weak conically central singularity.

A. Generalized Boulware-Deser collapse in five dimensions

It is important to note that the collapse dynamics of the Boulware-Deser spacetime do not change in the general setting, i.e., when the spacetime is radiating and inhomogeneous. If we allow the mass function to depend on both the temporal coordinate v and the radius of the star r [20,21], we have the metric

$$ds^2 = -f(v,\mathrm{r})dv^2 + 2dvd\mathrm{r} + \mathrm{r}^2(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2), \quad (20)$$

with

$$f(v,\mathrm{r}) = 1 + \frac{\mathrm{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16M(v,\mathrm{r})\alpha}{\mathrm{r}^4}}\right). \quad (21)$$

The energy momentum tensor for this two-component fluid distribution is given by

$$T_{ab} = \tilde{\mu} l_a l_b + (\tilde{\rho} + P)(l_a n_b + l_b n_a) + P g_{ab}, \quad (22)$$

where

$$l_a = \delta_a^0,$$
$$n_a = \frac{1}{2}\left[1 + \frac{\mathrm{r}^2}{4\alpha}\left(1 - \sqrt{1 + \frac{16M\alpha}{\mathrm{r}^4}}\right)\right]\delta_a^0 + \delta_a^1.$$

In the above $l_c l^c = n_c n^c = 0$ and $l_c n^c = -1$. The EGB field equations $\mathcal{G}^a{}_b = \kappa T^a{}_b$ are thus, in general, given by

$$\tilde{\mu} = \frac{3}{\kappa \mathrm{r}^3} M_v, \quad (23a)$$

$$\tilde{\rho} = \frac{3}{\kappa \mathrm{r}^3} M_{\mathrm{r}}, \quad (23b)$$

$$P = -\frac{1}{\kappa \mathrm{r}^2} M_{\mathrm{rr}}, \quad (23c)$$

with the three energy conditions

(i) The weak and strong energy conditions:

$$\tilde{\mu} \geq 0, \qquad \tilde{\rho} \geq 0, \qquad P \geq 0 \quad (\tilde{\mu} \neq 0). \quad (24)$$

(ii) The dominant energy condition:

$$\tilde{\mu} \geq 0, \qquad \tilde{\rho} \geq P \geq 0 \quad (\tilde{\mu} \neq 0). \quad (25)$$

Similar to the purely radiating case, there does exist an open set of mass functions $M(v,\mathrm{r})$ for which the continual gravitational contraction of the spacetime will terminate with the formation of a weak conical singularity at the center of the black hole. Added to this, there will be no violation of any of the energy conditions, and the singularity can possibly be resolved via an extension of the spacetime manifold. The trapping horizon is delayed in its formation due to the nonzero Gauss-Bonnet coupling constant α, however it eventually does form, covering the weak central conical singularity within the confines of a regular black hole.

V. HIGHER-DIMENSIONAL RADIATION SHELL COLLAPSE IN EGB GRAVITY

The collapse dynamics of null radiation differs with the presence of higher dimensions. It turns out that in dimensions $N > 5$, the gravitational contraction of the Boulware-Deser spacetime always terminates at a strong curvature singularity contained within a black hole. The N-dimensional purely radiating Boulware-Deser metric is written as

$$ds^2 = -f(v,\mathrm{r})dv^2 + 2dvd\mathrm{r} + \mathrm{r}^2 d\Omega_{N-2}^2, \quad (26)$$

with the $(N-2)$-sphere

$$d\Omega_{N-2}^2 = \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](d\theta^i)^2,$$

and where

$$f(v,\mathrm{r}) = 1 + \frac{\mathrm{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M(v)}{\mathrm{r}^{N-1}}\right)}\right). \quad (27)$$

In the above expression we have set $\hat{\alpha} = \alpha(N-3)(N-4)$ for convenience. The metric (26) is singular for all $N > 5$. The calculation of the Kretschmann invariant in general dimensions is a complicated endeavor, however we can perform a second-order.[3] Taylor expansion on (27) about α (where $\frac{\alpha M}{\mathrm{r}^{N-1}} \ll 1$) to acquire

$$f(v,\mathrm{r}) = 1 - \left(\frac{4}{N-3}\right)\frac{M(v)}{\mathrm{r}^{N-3}} + \left(\frac{16\hat{\alpha}}{(N-3)^2}\right)\frac{M(v)^2}{\mathrm{r}^{2N-4}}. \quad (28)$$

In dimensions $N = 6$ and $N = 7$, we found that $K \approx \mathrm{r}^{-20}$ and $K \approx \mathrm{r}^{-24}$, respectively, which is indicative of a very strong divergence; significantly faster than the five-dimensional case ($K \approx \mathrm{r}^{-4}$). Figures 3–5 depict the behavior of the various curvature invariants in dimensions $N = 6$ and $N = 7$. For these plots, we have chosen $\alpha = 2$ and $M(v) = 5$.

We also provide in Table I some key differences in the collapse of the Boulware-Deser spacetime between the case $N = 5$ and the cases $N > 5$.

The apparent horizon forms when

$$f(v,\mathrm{r}) = 0, \quad (29)$$

which implies that

[3]The reason we expand up to second order is that EGB gravity is a second-order theory, and so there is no real information loss with regards to the omission of higher-order terms.

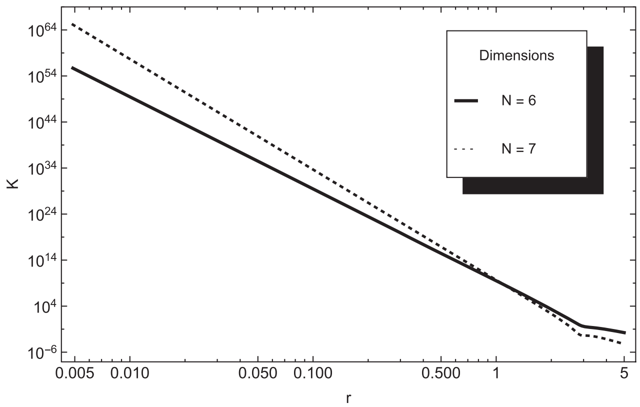

FIG. 3. Log-log plot showing the behavior of the Kretschmann scalar.

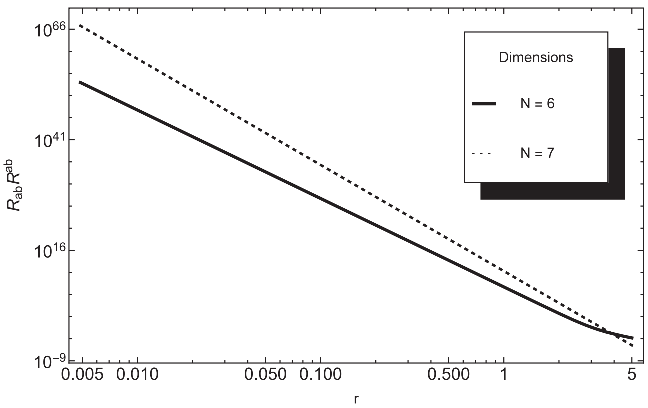

FIG. 4. Log-log plot indicating the behavior of the Ricci tensor squared.

$$1+\frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1-\sqrt{1+\frac{8\hat{\alpha}}{N-3}\left(\frac{2M(v)}{\mathsf{r}^{N-1}}\right)}\right)=0. \quad (30)$$

It is important to note that if the dimensions exceed five, $f(v,\mathsf{r})$ tends to infinity. The above equation has no explicit solution for r unless the dimension N is specified, however, we can write it as

$$\frac{4M(v)}{N-3}=\mathsf{r}^{N-5}(\hat{\alpha}+\mathsf{r}^2). \quad (31)$$

If we allow $M(v)=\lambda v$ as presented earlier, the above equation becomes

$$\frac{4\lambda v}{N-3}=\mathsf{r}^{N-5}(\hat{\alpha}+\mathsf{r}^2). \quad (32)$$

It is now clear that whenever $\mathsf{r}=0$, we will have $v=0$ unlike in the five-dimensional scenario. Therefore the apparent horizon begins to form at $\mathsf{r}=v=0$, for all $N>5$. The presence of the Gauss-Bonnet constant α has no affect on the collapse and further, does not delay the horizon formation, as was evident in the five-dimensional case. Also, when $\alpha=0$ this case mirrors the higher-dimensional general relativity collapse; gravitational

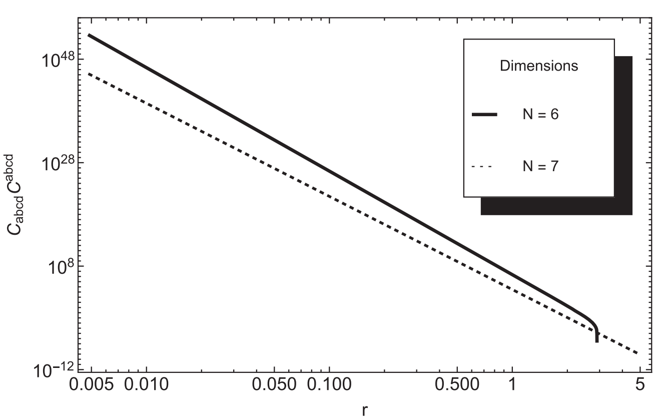

FIG. 5. Log-log plot demonstrating the divergence of the Weyl tensor squared.

TABLE I. Comparison between Boulware-Deser spacetimes.

Spacetime dimensions	$N=5$	$N>5$
Regularity	Yes	No
Kretschmann scalar	Divergent $K\approx\mathsf{r}^{-4}$	Divergent $K\approx\mathsf{r}^{-20}$ $(N=6)$ $K\approx\mathsf{r}^{-24}$ $(N=7)$
Other diffeomorphism invariants	All divergent	All divergent
Singularity existence	Yes	Yes
Singularity type	Conical	Curvature
Singularity strength	Weak	Very strong

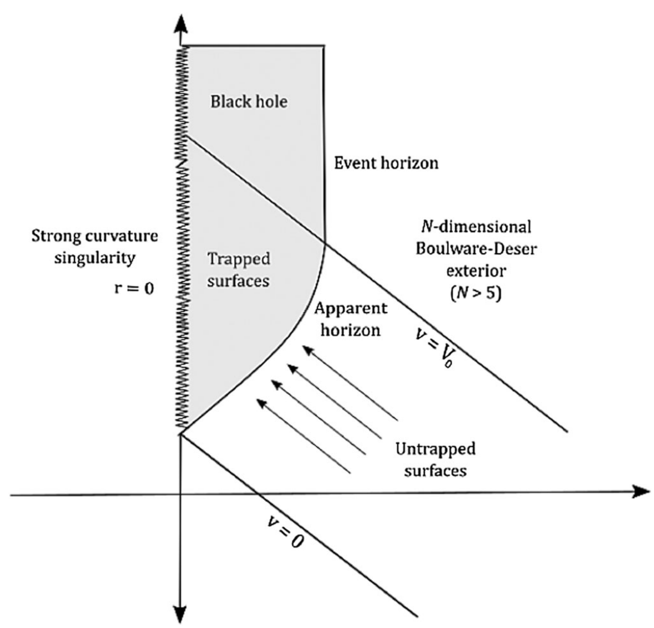

FIG. 6. Spacetime diagram of null shell collapse in N-dimensional EGB gravity.

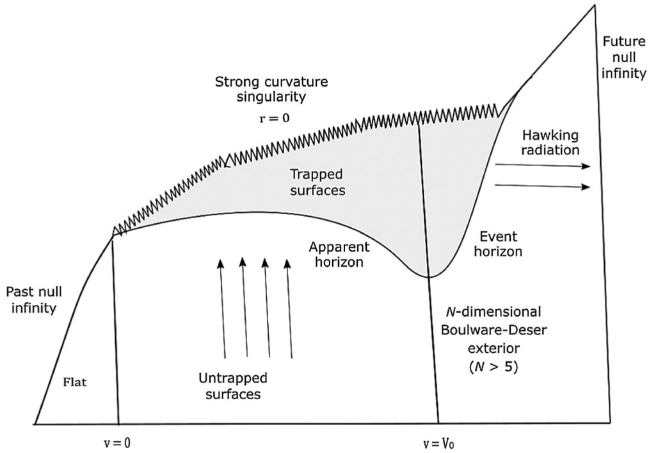

FIG. 7. Penrose diagram of null shell collapse in N-dimensional EGB gravity.

collapse ceases with a black hole containing a strong curvature singularity.

Figures 6 and 7 depict radiating matter falling into a singular black hole in ($N > 5$) dimensions. The apparent horizon forms immediately at $\mathsf{r} = v = 0$ (unlike in the five-dimensional case) and encloses trapped surfaces in a compact region for $0 < v < V_0$. At a time $v = V_0$ the apparent and event horizons match smoothly to form one trapping horizon at $\frac{4\lambda v}{N-3} = \mathsf{r}^{N-5}(\hat{\alpha} + \mathsf{r}^2)$ separating the exterior vacuum Boulware-Deser metric from the trapped surfaces within the black hole, inside of which is a strong central curvature singularity.

VI. COLLAPSE MODEL: SINGULARITY ANALYSIS

We will now examine the gravitational contraction of higher-dimensional matter and radiation described by the Boulware-Deser spacetime; a thick shell of type I and type II matter [so $M = M(v, \mathsf{r})$] collapses at the centre of symmetry in a universe which can be considered empty and asymptotically flat at great distances [6]. If K^a is as the tangent to nonspacelike geodesics where we have $K^a = \frac{dx^a}{dk}$ (k is the affine parameter), then $K^a{}_{;b}K^b = 0$ and

$$g_{ab}K^aK^b = \mathcal{B}, \tag{33}$$

where $\mathcal{B}$ is a constant which delineates different classes of geodesics. Timelike geodesics are characterized by $\mathcal{B} < 0$ while vanishing $\mathcal{B}$ is applicable to null geodesics. The quantities $\frac{dK^v}{dk}$ and $\frac{dK^{\mathsf{r}}}{dk}$ are calculated from the Euler-Lagrange equations

$$\frac{\partial L}{\partial x^a} - \frac{d}{dk}\left(\frac{\partial L}{\partial \dot{x}^a}\right) = 0, \tag{34}$$

where the Lagrangian is given by

$$L = \frac{1}{2}g_{ab}\dot{x}^a\dot{x}^b. \tag{35}$$

For the higher-dimensional Boulware-Deser spacetimes, these equations are given by

v-component:

$$\begin{aligned}\frac{dK^v}{dk} - \frac{1}{2}\Bigg[\frac{\mathsf{r}}{\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}}\right)}\right) \\ - \frac{\mathsf{r}^2}{4\hat{\alpha}}\left(\frac{8\hat{\alpha}}{N-3}\right)\left(\sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}}\right)}^{-1}\right) \\ \times \left(\left(\frac{2M'}{\mathsf{r}^{N-1}}\right) + 2\left(\frac{1-N}{\mathsf{r}^N}M\right)\right)\Bigg](K^v)^2 \\ + \mathsf{r}\sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](K^{\theta^i})^2 = 0.\end{aligned} \tag{36}$$

r-component:

$$\begin{aligned}\frac{dK^{\mathsf{r}}}{dk} - \left(\frac{8\hat{\alpha}}{N-3}\right)\left(\frac{\dot{M}}{2\hat{\alpha}\mathsf{r}^{N-3}}\right)(K^v)^2 \\ + \Bigg[\frac{\mathsf{r}}{\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}}\right)}\right) \\ - \frac{\mathsf{r}^2}{4\hat{\alpha}}\left(\frac{8\hat{\alpha}}{N-3}\right)\left(\sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M}{\mathsf{r}^{N-1}}\right)}^{-1}\right) \\ \times \left(\left(\frac{M'}{\mathsf{r}^{N-1}}\right) + 2\left(\frac{1-N}{\mathsf{r}^N}M\right)\right)\Bigg] \\ \times [f(v,\mathsf{r})(K^v)^2 - 2K^vK^{\mathsf{r}}] \\ + f(v,\mathsf{r})\mathsf{r}\sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](K^{\theta^i})^2 = 0,\end{aligned} \tag{37}$$

where $f(v, \mathsf{r})$ is given, as before, by

$$f(v,\mathsf{r}) = 1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M(v,\mathsf{r})}{\mathsf{r}^{N-1}}\right)}\right).$$

θ^i-components:

$$\frac{dK^{\theta^i}}{dk} + \frac{2}{\mathsf{r}}K^{\mathsf{r}}K^{\theta^i} + \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\cot(\theta^j)\right](K^{\theta^i})^2 = 0. \tag{38}$$

Following [5] we can write

$$K^v = \frac{P}{\mathsf{r}}, \tag{39}$$

where $P = P(v, \mathsf{r})$ is an arbitrary function. Noting that $\mathcal{B} = g_{ab}K^aK^b$, we have

$$K^v = \frac{dv}{dk} = \frac{P}{\mathsf{r}}, \tag{40a}$$

$$K^{\mathsf{r}} = \frac{dr}{dk} = f(v,\mathsf{r})\frac{P}{2\mathsf{r}} + \frac{\mathcal{B}\mathsf{r}}{2P} - \frac{l}{2\mathsf{r}P}. \tag{40b}$$

In the above, l is the impact parameter.

VII. CONDITIONS FOR A LOCALLY NAKED SINGULARITY

We now analyze how the final end state of gravitational collapse of the higher-dimensional Boulware-Deser spacetime is determined in terms of either a black hole or a naked singularity. The singularity forming as the final state of collapse will be naked if there are families of future-directed nonspacelike trajectories reaching observers sufficiently far away in spacetime, which then terminate in the past at the singularity. If no such trajectories exist and an event horizon forms at a time which is sufficiently early, the collapse terminates with a black hole. The equation for null geodesics for the spacetime (26) is given by the following

$$\frac{dv}{d\mathsf{r}} = \frac{2}{1 + \frac{\mathsf{r}^2}{2\hat{\alpha}}\left(1 - \sqrt{1 + \frac{8\hat{\alpha}}{N-3}\left(\frac{2M(v,\mathsf{r})}{\mathsf{r}^{N-1}}\right)}\right)}. \tag{41}$$

The above equation has a singularity $\forall N > 5$ at $v = 0$ and $\mathsf{r} = 0$, and its causal nature can be analyzed by utilizing the techniques associated with differential equation theory [22–24]. For the purpose of the following analysis, it is prudent to write Eq. (41) in the following form

$$\frac{dv}{d\mathsf{r}} = \frac{2}{1 - \left(\frac{4}{N-3}\right)\frac{M(v,\mathsf{r})}{\mathsf{r}^{N-3}} + \left(\frac{16\hat{\alpha}}{(N-3)^2}\right)\frac{M(v,\mathsf{r})^2}{\mathsf{r}^{2N-4}}}, \tag{42}$$

where we have made use of the earlier Taylor expanded form of the metric (28). Equation (42) can be written in the separable form

$$\frac{dv}{d\mathsf{r}} = \frac{A(v,\mathsf{r})}{C(v,\mathsf{r})}, \tag{43}$$

with the singularity at $v = \mathsf{r} = 0$, where the functions A and C vanish. If $v = \mathsf{r} = 0$, at the singularity, we can define the following

$$M_0 = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} M(v,\mathsf{r}), \tag{44a}$$

$$\dot{M}_0 = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} \frac{\partial}{\partial v} M(v,\mathsf{r}), \tag{44b}$$

$$M_0' = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} \frac{\partial}{\partial \mathsf{r}} M(v,\mathsf{r}), \tag{44c}$$

then Eq. (42) can be written, after a lengthy calculation, near the singularity point as

$$\frac{dv}{d\mathsf{r}} = \frac{(4N-8)\mathsf{r}^{2N-4}}{(2N-4)\mathsf{r}^{2N-4} - \left(\frac{4(N-1)}{N-3}\right)M_0\mathsf{r}^{N-1} + \left(\frac{32\hat{\alpha}}{(N-3)^2}M_0 - \frac{4}{N-3}\mathsf{r}^{N-1}\right)(M_0'\mathsf{r} + \dot{M}_0 v)}, \tag{45}$$

utilizing the techniques found in [8,22,24].

A. Existence of outgoing nonspacelike geodesics

If we let X be the tangent to the radial null geodesic, i.e., if we let X be a limiting value at $v = \mathsf{r} = 0$, the nature of this limiting value on a singular geodesic can be determined as

$$X_0 = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} X = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} \frac{v}{\mathsf{r}}. \tag{46}$$

If we use a suitable mass function, Eq. (45) and l'Hospital's rule, an explicit expression can be found for X_0 which will govern the behavior of the null geodesics in the vicinity of the singularity. This expression can be calculated as

$$X_0 = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} \frac{dv}{d\mathsf{r}} = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}} \left[\frac{(4N-8)\mathsf{r}^{2N-5}}{(2N-4)\mathsf{r}^{2N-5} - \left(\frac{4(N-1)}{N-3}\right)M_0\mathsf{r}^{N-2} + \left(\frac{32\hat{\alpha}}{(N-3)^2}M_0 - \frac{4}{N-3}\mathsf{r}^{N-1}\right)(M_0' + \dot{M}_0 X_0)}\right]. \tag{47}$$

Evaluation of the limits in the above equation it reduces significantly; at the singularity $v = \mathsf{r} = 0$ it becomes

$$X_0^2 + \frac{M_0'}{\dot{M}_0}X_0 = 0. \tag{48}$$

The nature of the singularity can hence be determined by analyzing the solution to the above algebraic equation which is valid for all spacetime dimensions greater than five.

B. Apparent horizon

The apparent horizon is one boundary of the trapped surface region within a given spacetime. As calculated earlier, it is defined for the higher-dimensional Boulware-Deser spacetime by

$$\frac{4M(v,\mathsf{r})}{N-3} = \mathsf{r}^{N-5}(\hat{\alpha}+\mathsf{r}^2). \tag{49}$$

Therefore, the slope of the apparent horizon is calculated in the following way

$$[2M]' = \left[(N-3)\frac{\hat{\alpha}}{2}\mathsf{r}^{N-5} + \frac{1}{2}(N-3)\mathsf{r}^{N-3}\right]', \tag{50a}$$

$$2\frac{\partial M}{\partial v}\left(\frac{dv}{d\mathsf{r}}\right)_{AH} + 2\frac{\partial M}{\partial \mathsf{r}} = \left[(N-3)\frac{\hat{\alpha}}{2}\mathsf{r}^{N-5} + \frac{1}{2}(N-3)\mathsf{r}^{N-3}\right]'. \tag{50b}$$

The slope of the apparent horizon at the central singularity is then given by

$$X_{AH} = \left(\frac{dv}{d\mathsf{r}}\right)_{AH} = \lim_{\substack{v\to 0\\ \mathsf{r}\to 0}}\left[\frac{\hat{\alpha}(N-3)(N-5)\mathsf{r}^{N-6}}{4\dot{M}_0} + \frac{(N-3)^2\mathsf{r}^{N-4} - 4M_0'}{4\dot{M}_0}\right]. \tag{51}$$

Evaluating the limits above, reduce the slope of the apparent horizon to the following

$$\left(\frac{dv}{d\mathsf{r}}\right)_{AH} = -\frac{M_0'}{\dot{M}_0}. \tag{52}$$

C. Sufficient conditions

We are now in a position to state the sufficient conditions for the existence of a locally naked central singularity for a collapsing Boulware-Deser spacetime for dimensions $N > 5$.

Proposition 1: Consider a collapsing generalized Boulware-Deser spacetime from a regular epoch with dimensions $N > 5$, and a mass function $M(v,\mathsf{r})$ satisfying all physically reasonable energy conditions, and is at least C^2. If the following are satisfied:

(1) The limits of the partial derivatives of the mass function $M(v,\mathsf{r})$ exist at the central singularity, and obey the conditions:

$$\frac{32\hat{\alpha}}{(N-3)^2}M_0M_0' \geq 0, \qquad \dot{M}_0 > 0,$$

(2) There exist one or more roots (positive and real) X_0 to the equation

$$X_0^2 + \frac{M_0'}{\dot{M}_0}X_0 = 0,$$

(3) At least one of the positive and real roots is less than

$$\left(\frac{dv}{d\mathsf{r}}\right)_{AH} = -\frac{M_0'}{\dot{M}_0},$$

at the central singularity,

then the singularity is locally naked with outgoing C^1 radial null geodesics escaping to the future.

VIII. COSMIC CENSORSHIP

Utilizing the results of the previous sections we will now show that a naked singularity *may* or *may not* be possible in any dimension greater than five for the generalized Boulware-Deser spacetime. In order to demonstrate this we consider (48), which is a second order algebraic equation in X_0. It admits two solutions, namely the trivial solution

$$X_0 = 0,$$

and

$$X_0 = -\frac{M_0'}{\dot{M}_0}.$$

The latter solution may be valid if one of $\dot{M}_0$ or M_0' is negative (in which case, M_0 itself is negative), which cannot happen, by point 1 in *Proposition 1*. Thus, the solution is real but negative and can only validate *Proposition 1* if the original mass function is negative, which is impossible for any spacetime containing matter. Therefore, for all $N > 5$ and for any positive and real mass function $M(v,\mathsf{r})$, *Proposition 1* will never be satisfied. With this, it is not possible to determine whether naked singularities are possible in any dimensions higher than five, since the conditions stated in *Proposition 1* are only sufficient conditions.

This scenario differs from the general relativity counterpart since the dimensions here, play no role in how the collapse ensues and terminates; the above solutions are independent of dimensions entirely and hold for all $N > 5$. This is another fundamental distinction between EGB gravity and conventional relativity.

We can finally use the above to state the following:

Consider a collapsing generalized N-dimensional Boulware-Deser spacetime from a regular epoch with $N > 5$, and a positive and real mass function $M(v,\mathsf{r})$ satisfying all physically reasonable energy conditions, and is at least C^2 (or equivalently C^∞). The final outcome of gravitational collapse is a strong curvature singularity which may be covered within the confines of a stellar-mass black hole.

IX. HIGHER-ORDER LOVELOCK GRAVITY

Higher order theories of gravity have been the subject of much study. To modify general relativity, nonlinear forms of the Riemann and Ricci tensor, and the Ricci scalar were introduced. The second order equations of motion resulting from linear forms is advantageous in four dimensions; however as shown by Lovelock [25,26] it is possible to introduce a polynomial form of the Lagrangian which is of quadratic order (EGB gravity) or cubic order and so on. Static solutions for black holes in higher-order theories were obtained by Charmousis [27]. The action in Lovelock gravity is given by

$$S = \int d^N x \sqrt{-g} \sum_{k=0}^{N/2} \alpha_k \mathcal{R}^k + S_{\mathrm{matter}}, \tag{53}$$

where we have

$$\mathcal{R}^k = \frac{1}{2^k} \delta^{c_1 d_1 \ldots c_k d_k}_{a_1 b_1 \ldots a_k b_k} \prod_{r=1}^{k} R^{a_r b_r}{}_{c_r d_r},$$

and $\delta^{c_1 d_1 \ldots c_k d_k}_{a_1 b_1 \ldots a_k b_k}$ is the Kronecker delta. In this paper, we have considered Einstein-Gauss-Bonnet gravity, which is second-order Lovelock gravity. If we now consider third-order Lovelock gravity, the above action (53) reduces to

$$S = \int d^N x \sqrt{-g} (\alpha_0 + \alpha_1 \mathcal{R} + \alpha_2 \mathcal{R}^2 + \alpha_3 \mathcal{R}^3), \tag{54}$$

where α_0 is the cosmological term, α_1 is the constant (usually unity) associated with the Einstein-Hilbert action ($\mathcal{R} = R$), α_2 and α_3 are constants associated with the second order (Gauss-Bonnet) and third order Lovelock terms. In the above, $\mathcal{R}^2 = L_{GB}$ as before and

$$\begin{aligned} \mathcal{R}^3 = {} & R^3 + 2R^{abcd} R_{cdef} R^{ef}{}_{ab} \\ & + 8R^{ab}{}_{ce} R^{cd}{}_{bf} R^{ef}{}_{ad} + 24 R^{abcd} R_{cdbe} R^{e}{}_{a} \\ & + 3RR^{abcd} R_{cdab} + 24 R^{abcd} R_{ca} R_{db} \\ & + 16 R^{ab} R_{bc} R^{c}{}_{a} - 12 RR^{ab} R_{ab}, \end{aligned} \tag{55}$$

is the third-order Lovelock Lagrangian. Varying the action with respect to the metric g_{ab} gives the Einstein-Gauss-Bonnet-Lovelock field equations

$$G^E_{ab} + \alpha_2 H^{GB}_{ab} + \alpha_3 H^{(3)}_{ab} = T_{ab}, \tag{56}$$

where G^E_{ab} is the Einstein tensor, H^{GB}_{ab} is the Lanczos tensor given by (12) and

$$\begin{aligned} H^{(3)}_{ab} = {} & -3(4R^{fecd} R_{cdge} R^{g}{}_{bfa} \\ & - 8R^{fe}{}_{gc} R^{cd}{}_{fa} R^{g}{}_{bed} + 2R_b{}^{fcd} R_{cdge} R^{ge}{}_{fa} \\ & - R^{fecd} R_{cdfe} R_{ba} + 8R^{f}{}_{bce} R^{cd}{}_{fa} R^{e}{}_{d} \\ & + 8R^{c}{}_{bfd} R^{fe}{}_{ca} R^{d}{}_{e} + 4R_b{}^{fcd} R_{cdae} R^{e}{}_{f} \\ & - 4R_b{}^{fcd} R_{cdfe} R^{e}{}_{a} + 4R^{fecd} R_{cdfa} R_{be} \\ & + 2RR_b{}^{dfe} R_{feda} + 8R^{f}{}_{bae} R^{e}{}_{c} R^{e}{}_{f} \\ & - 8R^{c}{}_{bfe} R^{f}{}_{c} R^{e}{}_{a} - 8R^{fe}{}_{ca} R^{c}{}_{f} R_{be} \\ & - 4RR^{f}{}_{bae} R^{e}{}_{f} + 4R^{fe} R_{ef} R_{ba} \\ & - 8R^{f}{}_{b} R_{fe} R^{e}{}_{a} + 4RR_{be} R^{e}{}_{a} \\ & - R^2 R_{ba}) - \frac{1}{2} \mathcal{R}^3 g_{ab}. \end{aligned} \tag{57}$$

The nontriviality of the above expression requires that the dimension of the spacetime in third-order Lovelock gravity has to satisfy $N \geq 7$. The field equations of third-order Lovelock gravity in seven dimensions are the most general second-order differential equations which produce the solutions of gravity. For orders of four (and higher), the field equations will cease to be second-order.

Asymptotically flat, static solutions which represented black holes with inner and outer horizons were found by Dehghani and Shamirzaie [28] in third-order Lovelock gravity. They found that these solutions do not exist in general relativity and Einstein-Gauss-Bonnet gravity and were only prevalent in this theory. They further computed the entropy, temperature as well as other quantities, including the mass of the black hole solutions. Ghosh *et al.* [29] studied black hole solutions and their temperature in a cloud string back ground in Einstein, Einstein-Gauss-Bonnet and third-order Lovelock gravity. The presence of the higher-order curvature corrections greatly affects the thermodynamics of the black hole solutions. It must be emphasized that these solutions were also static solutions and, as it stands, there are no radiating solutions of any kind in third-order Lovelock gravity. The collapse analysis of radiating shells undertaken in the earlier sections of this paper could, in principle, be possible should a radiating solution be found in third-order Lovelock gravity.

X. DISCUSSION

In this paper, we extended the analysis of radiating shell collapse in five-dimensional EGB gravity [19] to arbitrary dimensions. We first provided brief descriptions of five-dimensional collapse in general relativity and EGB gravity. We then found that in higher dimensions, the collapse of the radiating Boulware-Deser spacetime ceases with a strong central curvature singularity, a fundamentally different outcome from the five-dimensional case. Various plots were given for various curvature scalars and a table was provided to show the comparison between five-dimensional and higher-dimensional contraction. A singularity analysis was undertaken in full and sufficient conditions for the formation of a naked singularity were developed. We then showed that these conditions can never be satisfied for any positive and real mass function, and therefore, it is not possible to show that a naked singularity forms upon the cessation of gravitational collapse. This is fundamentally different to the higher-dimensional general relativity cases, where the dimensions themselves affected the dynamics and the end states of collapse.

We can therefore conclude that the notion of higher-order curvature indicative of a modified gravitation theory like EGB gravity, plays a significant role in the dynamics as well as the gravitational collapse of a radiating spacetime. In this case, the nature of cosmic censorship is affected by higher-order theories.

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