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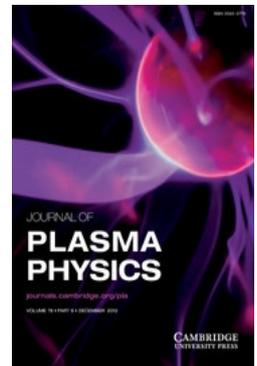
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# Linear electrostatic waves in two-temperature electron–positron plasmas

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**Abstract.** Linear electrostatic waves in a magnetized four-component, two-temperature electron–positron plasma are investigated, with the hot species having the Boltzmann density distribution and the dynamics of cooler species governed by fluid equations with finite temperatures. A linear dispersion relation for electrostatic waves is derived for the model and analyzed for different wave modes. Analysis of the dispersion relation for perpendicular wave propagation yields a cyclotron mode with contributions from both cooler and hot species, which in the absence of hot species goes over to the upper hybrid mode of cooler species. For parallel propagation, both electron-acoustic and electron plasma modes are obtained, whereas for a single-temperature electron–positron plasma, only electron plasma mode can exist. Dispersion characteristics of these modes at different propagation angles are studied numerically.

## 1. Introduction

The earlier theoretical studies on linear waves in electron–positron plasmas have largely focussed on the relativistic regime relevant to astrophysical contexts (Yu et al. 1984; Lakhina and Verheest 1997; Lontano et al. 2001; Fonseca et al. 2003; Matsukiyo and Hada 2003; Machabeli et al. 2005; Nishikawa et al. 2006). This is largely due to the fact that the production of these electron–positron pairs require high-energy processes, which are more common in astrophysical conditions such as those which exist in the environments of pulsars, active galactic nuclei, gamma-ray bursts, solar flares, and black holes. The majority of the reported studies have been primarily limited to single-temperature electron–positron plasmas.

However, it is plausible that non-relativistic astrophysical electron–positron plasmas may exist, given the effect of efficient cooling by cyclotron emissions (Zank and Greaves 1995; Bhattacharyya et al. 2003). In laboratory plasmas non-relativistic electron–positron plasmas can be created by using two different schemes. In one scheme, a relativistic electron beam when impinges on high  $Z$ -target produces positrons in abundance. The relativistic pair of electrons and positrons is then trapped in a magnetic mirror and cools down rapidly by radiation, thus producing non-relativistic pair plasmas (Trivelpiece 1972). In another scheme positrons can be accumulated

from a radioactive source (Surko et al. 1989). Such non-relativistic electron–positron plasmas have been produced in the laboratory by many researchers (Greaves et al. 1994; Greaves and Surko 1995; Liang et al. 1998).

This has given an impetus to many theoretical works on non-relativistic electron–positron plasmas (Stewart and Laing 1992; Iwamoto 1993; Zank and Greaves 1995; Zhao et al. 1996; Bhattacharyya et al. 2003; Lazarus et al. 2008; Saeed and Mushtaq 2009; Kourakis and Saini 2010). Stewart and Laing (1992) studied the dispersion properties of linear waves in equal-mass plasmas and found that due to the special symmetry of such plasmas, well known phenomena such as Faraday rotation and whistler wave modes disappear. Iwamoto (1993) studied the collective modes in non-relativistic electron–positron plasmas using the kinetic approach. He found that the dispersion relations for longitudinal modes in electron–positron plasma for both unmagnetized and magnetized electron–positron plasmas were similar to the modes in one-component electron or electron–ion plasmas. The transverse modes for the unmagnetized case were also found to be similar. However, the transverse modes in the presence of a magnetic field were found to be different from those in electron–ion plasmas. In an electron–ion plasma, the extraordinary wave is known to have two cutoff frequencies. However, the mode is found to have just one such cutoff in an electron–positron plasma. Moreover, the hybrid

\* Deceased

resonances present in the former are not found in an electron–positron plasma.

Studies of wave propagation in electron–positron plasmas continue to highlight the role played by the equal mass of electrons and positrons. For example, the low-frequency ion acoustic wave, a feature of electron–ion plasmas due to significantly different masses of electrons and ions, has no counterpart in an electron–positron plasma. In one such study, using the two-fluid model with a single temperature of positrons and electrons, Zank and Greaves (1995) investigated linear and nonlinear longitudinal and transverse electrostatic and electromagnetic waves in a non-relativistic electron–positron plasma, both in the absence and presence of an external magnetic field. They found that several of the modes present in electron–ion plasmas also existed in electron–positron plasmas, but in a modified form because of the symmetry derived from the common mass of electrons and positrons. On the other hand, it is noted that the whistler and lower hybrid modes are non-existent in electron–positron plasmas. A study of two-stream instability yielded similar results to the electron–ion case except that the growth rate was now substantially larger due to equality in the masses of electrons and positrons. In their nonlinear analysis, solitary waves are found to exist in the subsonic regime, and the width of the soliton was found to be proportional to the wave speed, while in electron–ion plasmas, the amplitude is related to the wave speed. Esfandyari-Kalejahi et al. (2006) studied oblique modulation of electrostatic modes and envelope excitations in pair-ion and electron–positron plasmas. Their investigation showed the existence of two distinct linear electrostatic modes, namely an acoustic lower mode and the Langmuir-type, optic-type upper mode. Shukla and Shukla (2007) derived a new dispersion relation for low-frequency electrostatic waves in strongly magnetized non-uniform electron–positron plasma. They showed that the dispersion relation admits a new purely growing instability in the presence of equilibrium density and magnetic field inhomogeneities.

In astrophysical and cosmic plasmas a minority of cold electrons and heavy ions exist along with hot electron–positron pairs (Berezhiani and Mahajan 1995). For example, an outflow of the electron–positron plasma from pulsars on entering into an interstellar cold, low-density electron–ion plasma can lead to the formation of two-temperature multi-species electron–positron–ion plasma (Shatashvili et al. 1997). Shatashvili et al. (1997) studied the modulational interactions of electromagnetic and sound waves in hot electron–positron unmagnetized plasma with small fraction of cold electron–ion plasma. Also, possibility of soliton formation in such plasmas is investigated. Positron-acoustic solitary waves have been studied in four-component unmagnetized plasma with cold positrons, immobile positive ions, and the Boltzmann distributed electrons and positrons (Tribeche et al. 2009).

In pulsar magnetosphere, a possibility for the co-existence of two types of cold and hot electron–positron populations has been suggested by Bharuthram (1992). This is inspired by the Sturrock (1971) model where an accelerated electron moving on curved magnetic field line emits curvature photon, which produces an electron–positron pair. These secondary particles can produce curvature radiation that will produce new pairs and so on. Therefore, both electron and positron populations can be subdivided in two group of distinct temperatures, one modeling the original plasma, and another the higher-energy cascade-bred pairs. Such distinct population should coexist on a timescale shorter than the thermalization time of species. Large-amplitude solitons in electron–positron plasmas having equal hot and cold components of both species have been studied by a number of authors (Pillay and Bharuthram 1992; Verheest et al. 1996; Lazarus et al. 2008).

To our knowledge, no work has been done on the properties of the linear electrostatic modes in magnetized electron–positron plasma having equal cold and hot components of both species. In this paper we extend the work of Zank and Greaves (1995) on a single-temperature-magnetized two-component electron–positron plasma to a magnetized four-component, two-temperature plasma having hot and cool electrons and positrons. We neglect the effects due to electron–positron pair annihilation, which usually occurs at much longer characteristic time scales compared with the time in which the collective interaction between the charged particles takes place (Surko and Murphy 1990). Further, the linear electrostatic waves studied here in a simple fluid model can not handle the possible Landau damping of modes. However, the modes having phase velocities away from thermal velocities of either hot or cold species are not expected to suffer significant Landau damping.

The paper is structured as follows. In Sec. 2, the basic equations for electron–positron plasma are presented and the linear modes for arbitrary angle of propagation as well as the two extreme limits, viz. perpendicular and parallel propagation, are discussed. In Sec. 3, we present the numerical analysis. Our results are concluded in Sec. 4.

## 2. Basic theory

To study the linear electrostatic modes, we consider a homogeneous magnetized, four-component electron–positron plasma consisting of cool electrons and cool positrons with equal temperatures and equilibrium number densities denoted by  $T_c$  and  $n_{0c}$ , respectively, and hot electrons and hot positrons with equal temperatures and equilibrium number densities denoted by  $T_h$  and  $n_{0h}$ , respectively. Here temperatures are expressed in energy units and wave propagation is taken in  $x$ -direction at an angle  $\theta$  to the ambient magnetic field  $\mathbf{B}_0$ , which is assumed to be in the  $x$ – $z$  plane. The hot isothermal species are assumed to be unmagnetized and have the

Boltzmann distribution. Their number densities can be written as

$$n_{eh} = n_{0h} \exp\left(\frac{e\phi}{T_h}\right) \quad (2.1)$$

and

$$n_{ph} = n_{0h} \exp\left(\frac{-e\phi}{T_h}\right), \quad (2.2)$$

where  $n_{eh}$  ( $n_{ph}$ ) is the number density of the hot electrons (positrons) and  $\phi$  is the electrostatic potential. The assumption of Boltzmann distribution of hot electrons and positrons is justified provided they have sufficiently high temperatures, much greater than that of cooler species such that their thermal velocities parallel to the magnetic field exceed the phase velocity of the modes so that they are able to establish the Boltzmann distribution. However, treating the hot species as unmagnetized is justified when the perturbation wavelengths are shorter than their gyroradii such that both hot electrons and positrons follow essentially straight line orbits across the magnetic field direction. In such situations the magnetic field effects on hot species are not felt. The dynamics of cooler isothermal species are governed by fluid equations, namely

the continuity equations,

$$\frac{\partial n_{jc}}{\partial t} + \nabla \cdot (n_{jc} \mathbf{v}_{jc}) = 0, \quad (2.3)$$

the equations of motion,

$$\frac{\partial \mathbf{v}_{jc}}{\partial t} + \mathbf{v}_{jc} \cdot \nabla \mathbf{v}_{jc} = -\frac{\epsilon_j e}{m} [\nabla \phi - \mathbf{v}_{jc} \times \mathbf{B}_0] - \frac{T_c}{n_{jcm}} \nabla n_{jc}, \quad (2.4)$$

where  $\epsilon_j = +1(-1)$  for positrons (electrons) and  $j = e(p)$  for the electrons (positrons). The system is closed by the Poisson equation

$$\epsilon_0 \frac{\partial^2 \phi}{\partial x^2} = -e(n_{pc} - n_{ec} + n_{ph} - n_{eh}). \quad (2.5)$$

In the above equations,  $n_j$  and  $\mathbf{v}_j$  are the number densities and fluid velocities, respectively, of the  $j$ th species. It must be noted that the chosen plasma model is an extension of that used by Zank and Greaves (1995). Here the two additional hot species having Boltzmann density distributions have been included.

To determine the linear dispersion relation, (2.1)–(2.5) are linearized. For perturbations varying as  $\exp(i(kx - \omega t))$ ,  $\partial/\partial t$  is replaced with  $-i\omega$  and  $\partial/\partial x$  with  $ik$  ( $k$  and  $\omega$  being the wave number and the frequency of the wave, respectively). Hence, the perturbed number densities for electrons and positrons become, respectively,

$$n_{ec} = -\frac{n_{0c} e k^2 (\omega^2 - \Omega^2 \cos^2 \theta) \phi}{m(\omega^4 - \omega^2(k^2 v_{tc}^2 + \Omega^2) + k^2 v_{tc}^2 \Omega^2 \cos^2 \theta)}, \quad (2.6)$$

and

$$n_{pc} = \frac{n_{0c} e k^2 (\omega^2 - \Omega^2 \cos^2 \theta) \phi}{m(\omega^4 - \omega^2(k^2 v_{tc}^2 + \Omega^2) + k^2 v_{tc}^2 \Omega^2 \cos^2 \theta)}. \quad (2.7)$$

Here  $v_{tc} = (T_c/m)^{1/2}$  is the thermal speed of cooler electrons or positrons, and  $\Omega = eB_0/m$  is the cyclotron frequency of electrons or positrons irrespective of whether they belong to cooler or hot population. From (2.1) and (2.2), the perturbed densities for the hot species are given by

$$n_{eh} = n_{0h} \frac{e\phi}{T_h} \quad (2.8)$$

and

$$n_{ph} = -n_{0h} \frac{e\phi}{T_h}. \quad (2.9)$$

Substituting (2.6)–(2.9) into the Poisson's equation (2.5), the general dispersion relation for the two-temperature electron–positron plasma is found to be

$$\omega^2(\omega^2 - \Omega^2) - \left(k^2 v_{tc}^2 + \frac{2\omega_{pc}^2 k^2 v_{th}^2}{2\omega_{ph}^2 + k^2 v_{th}^2}\right) (\omega^2 - \Omega^2 \cos^2 \theta) = 0, \quad (2.10)$$

where  $v_{tj} = (T_j/m)^{1/2}$  and  $\omega_{pj} = (n_{0j} e^2 / \epsilon_0 m)^{1/2}$  are the thermal speed and the plasma frequency of the  $j$ th species ( $j = c, h$  for cool and hot species of electrons or positrons). For wave frequencies much lower than the cyclotron frequency, satisfying  $\omega \ll \Omega \cos \theta$  and  $T_c \ll T_h$ , (2.10) yields an electron-acoustic (or positron-acoustic) mode,

$$\omega^2 \approx \frac{k^2 v_{ea}^2 \cos^2 \theta}{1 + \frac{1}{2} k^2 \lambda_{dh}^2}, \quad (2.11)$$

where  $v_{ea} = (n_{c0}/n_{h0})^{1/2} v_{th}$  is the electron-acoustic speed, and  $\lambda_{dh} = v_{th}/\omega_{ph}$  is the hot electron Debye length. This mode is similar to the one discussed by Tokar and Gary (1984) in a magnetized plasma consisting of two-electron temperature populations and ions.

In the absence of hot species (hot electrons and positrons), the dispersion relation (2.10) reduces to

$$\omega^4 - \omega^2(k^2 v_{tc}^2 + \omega_{UH}^2) + (k^2 v_{tc}^2 + 2\omega_{pc}^2) \Omega^2 \cos^2 \theta = 0, \quad (2.12)$$

where

$$\omega_{UH}^2 = \Omega^2 + 2\omega_{pc}^2 \quad (2.13)$$

is the upper hybrid frequency associated with cooler species (Zank and Greaves 1995). In order to gain physical insight into the solution of the dispersion relation, the two extreme limits of (2.10) will now be considered, viz. pure perpendicular and pure parallel propagations.

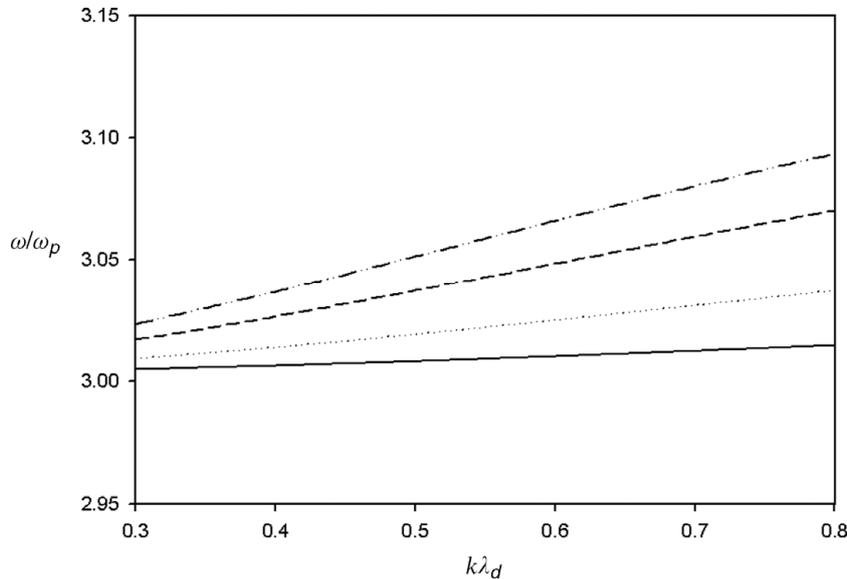
### 2.1. Perpendicular propagation

Considering the perpendicular ( $\theta = 90^\circ$ ) limit, the general dispersion relation (2.10) reduces to:

$$\omega^4 - \omega^2 \left( \Omega^2 + k^2 v_{tc}^2 + \frac{2\omega_{pc}^2 k^2 v_{th}^2}{2\omega_{ph}^2 + k^2 v_{th}^2} \right) = 0. \quad (2.14)$$

Hence, the normal mode frequencies are

$$\omega = 0, \quad (2.15)$$



**Figure 1.** Normalized real frequency as a function of the normalized wavenumber. The fixed parameters are  $R = 0.333$ ,  $T_c/T_h = 0.01$ , and  $\theta = 90^\circ$ . The parameter labelling the curves is the equilibrium density ratio  $n_{0c}/n_0 = 0.1$  (solid),  $0.3$  (dotted),  $0.5$  (broken), and  $0.6$  (dashdotdot).

which is a non-propagating mode, and

$$\omega^2 = \Omega^2 + k^2 v_{ic}^2 + \frac{2\omega_{pc}^2 k^2 v_{th}^2}{2\omega_{ph}^2 + k^2 v_{th}^2}. \quad (2.16)$$

This is the electron cyclotron mode in the electron-positron plasma with contributions from both the thermal motion of the adiabatic cooler species and the acoustic motion due to two species at different temperatures. The electron cyclotron mode here is modified due to the presence of hot species. The last term on the right-hand side of (2.16) is a modification to the result obtained by Zank and Greaves (1995) (refer to their Table 1).

When hot species are not present, i.e.,  $n_{0h} = 0$  ( $\omega_{ph} = 0$ ), we obtain from (2.16),

$$\omega^2 = \omega_{UH}^2 + k^2 v_{ic}^2. \quad (2.17)$$

which is a upper hybrid mode similar to the one obtained by Zank and Greaves (1995) (refer to their Table 1).

### 2.2. Parallel propagation

Considering the limit of parallel propagation ( $\theta = 0^\circ$ ), the general dispersion relation (2.10) reduces to

$$(\omega^2 - \Omega^2) \left( \omega^2 - k^2 v_{ic}^2 - \frac{2\omega_{pc}^2 k^2 v_{th}^2}{2\omega_{ph}^2 + k^2 v_{th}^2} \right) = 0. \quad (2.18)$$

From the dispersion relation (2.18), it can be seen that the two modes, i.e., the electron cyclotron mode ( $\omega = \Omega$ ) and electron-acoustic mode, are decoupled. The parallel propagating electron acoustic mode is given by

$$\omega^2 = k^2 v_{ic}^2 + \frac{2\omega_{pc}^2 k^2 v_{th}^2}{2\omega_{ph}^2 + k^2 v_{th}^2}. \quad (2.19)$$

In the absence of hot species, (2.19) reduces to

$$\omega^2 = k^2 v_{ic}^2 + 2\omega_{pc}^2, \quad (2.20)$$

which is a electron plasma mode arising from the motion of cooler species. Hence, it is clear that the electron-acoustic mode given by (2.19) exists due to the presence of hot electron (positron) species.

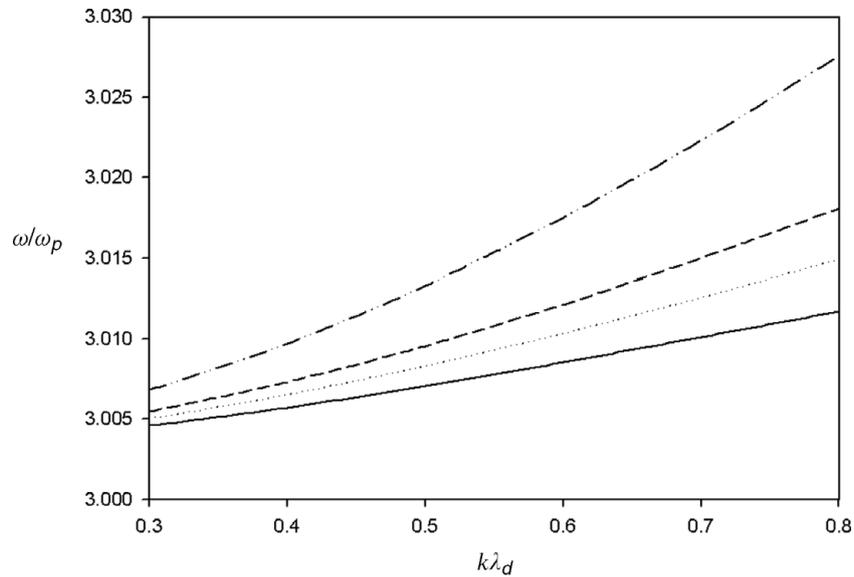
### 3. Numerical results

To obtain numerical results, normalized form of (2.10) is used. Normalizations used are as follows: the fluid speeds are normalized with the thermal velocity  $v_{th} = (T_h/m)^{1/2}$  of hot species, the particle density by the total equilibrium plasma density  $n_0 = n_{0c} + n_{0h}$ , the temperatures by  $T_h$ , the spatial length by  $\lambda_d = (\epsilon_0 T_h / n_0 e^2)^{1/2}$ , and the time by  $\omega_p^{-1} = (n_0 e^2 / \epsilon_0 m)^{-1/2}$ . The normalized general dispersion relation can be written as

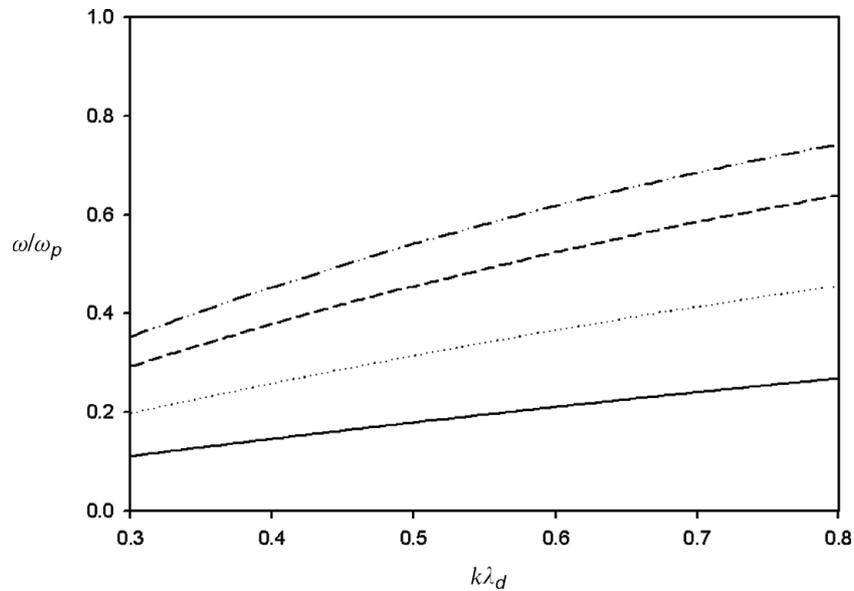
$$\omega'^4 - \omega'^2 \left( \frac{1}{R^2} + k'^2 \frac{T_c}{T_h} + \frac{k'^2 n'_{0c}}{n'_{0h} + \frac{1}{2} k'^2} \right) + \frac{\cos^2 \theta}{R^2} \left( k'^2 \frac{T_c}{T_h} + \frac{k'^2 n'_{0c}}{n'_{0h} + \frac{1}{2} k'^2} \right) = 0, \quad (3.1)$$

where  $\omega' = \omega/\omega_p$ ,  $k' = k\lambda_d$ ,  $n'_{0h} = n_{0h}/n_0$ ,  $n'_{0c} = n_{0c}/n_0$ , and  $R = \omega_p/\Omega$  is a ratio of plasma frequency to cyclotron frequency. The effect of propagation angle, density, and temperature of hot and cool electrons and positrons on wave modes are studied here.

First, we investigate the waves propagating perpendicular to the ambient magnetic field by using the general dispersion relation (2.10). Figure 1 shows the variation of normalized real frequency with normalized wavenumber ( $k\lambda_d$ ) for different values of the normalized cold species density ratios ( $n_{0c}/n_0$ ) for other fixed parameters,  $R = 0.333$ ,  $T_c/T_h = 0.01$ , and  $\theta = 90^\circ$ . Figures 1–6, are plotted for  $k\lambda_d$  range restricted to 0.3–0.8 so that all our assumptions, namely  $k\lambda_{dh} < 1$ ,  $k\lambda_{dc} < 1$ , and  $kv_{th}/\Omega > 1$ ,



**Figure 2.** Normalized real frequency as a function of the normalized wavenumber. The fixed plasma parameters are  $R = 0.333$ ,  $n_{0c}/n_0 = 0.1$ , and  $\theta = 90^\circ$ . The parameter labelling the curves is the temperature ratio  $T_c/T_h = 0.0$  (solid),  $0.01$  (dotted), and  $0.05$  (broken).



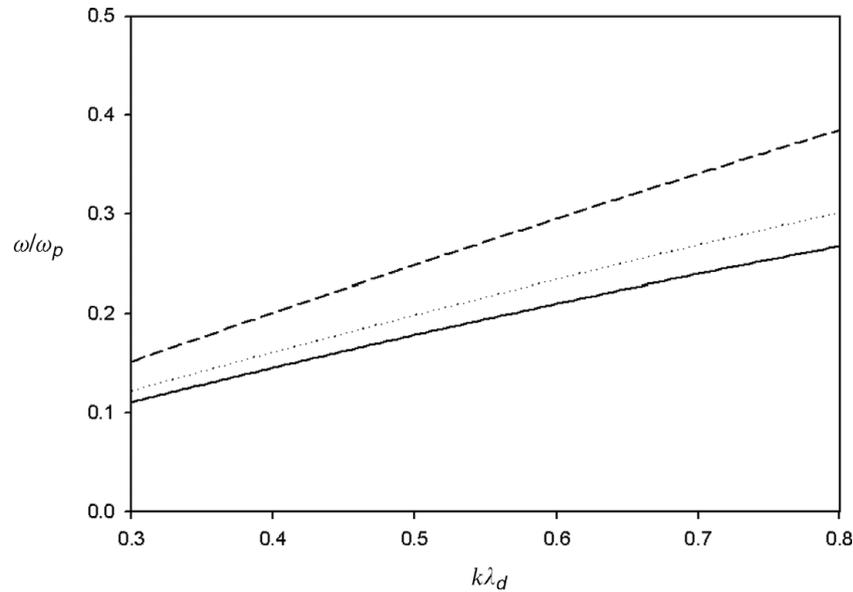
**Figure 3.** Normalized real frequency as a function of the normalized wavenumber. The fixed parameters are  $R = 0.333$ ,  $T_c/T_h = 0.01$ , and  $\theta = 0^\circ$ . The parameter labelling the curves is the equilibrium density ratio  $n_{0c}/n_0 = 0.1$  (solid),  $0.3$  (dotted),  $0.5$  (broken), and  $0.6$  (dashdot).

are satisfied. The frequency increases with increase in cold species density. This is the cyclotron mode modified by the presence of finite temperature cold species (second term in (2.16)) and acoustic term arising due to the presence of cold and hot species (last term in (2.16)). At large values of  $(n_{0c}/n_0)$ , i.e., for small quantity of hot species, the mode approaches upper hybrid frequency ((2.17)).

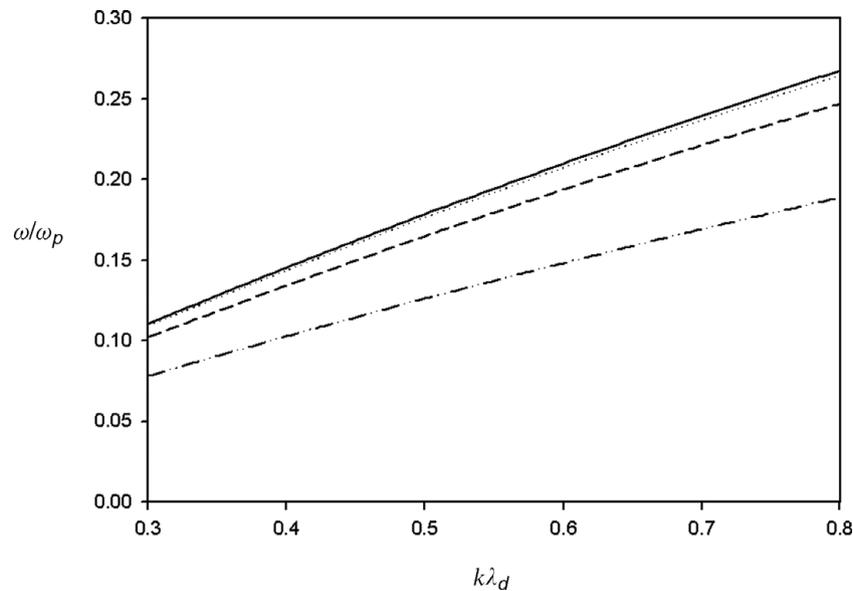
Figure 2 shows the variation of normalized real frequency with normalized wavenumber ( $k\lambda_d$ ) for different values of normalized cold to hot species temperature ratios ( $T_c/T_h$ ) for other fixed parameters,  $R = 0.333$ ,  $n_{0c}/n_0 = 0.1$ , and  $\theta = 90^\circ$ . This is the electron cyclotron

mode, as the curve is observed close to  $\omega/\omega_p = 1/R \approx 3$  and mode frequency increases with increase in  $T_c/T_h$  values. It must be emphasized that the behavior of the curves in Fig. 2 is a characteristic of a four-component, two-temperature electron–positron plasma and has not been reported in the literature before.

Figure 3 shows normalized frequency as a function of normalized wavenumber for parallel propagation  $\theta = 0^\circ$  for various values of cold species density for the parameters of Fig. 1. It is noted that the frequency of the mode increases with increase in  $n_{0c}/n_0$ . From dispersion curves, the mode is identified as an electron-acoustic mode (cf. (2.19)). This is a feature of the



**Figure 4.** Normalized real frequency as a function of the normalized wavenumber. The fixed plasma parameters are  $R = 0.333$ ,  $n_{0c}/n_0 = 0.1$ , and  $\theta = 0^\circ$ . The parameter labelling the curves is the temperature ratio  $T_c/T_h = 0.01$  (solid),  $0.02$  (dotted), and  $0.05$  (broken).



**Figure 5.** Normalized real frequency as a function of the normalized wavenumber showing the electron-acoustic branch for various angles of propagation  $\theta = 0^\circ$  (solid),  $9^\circ$  (dotted),  $22.5^\circ$  (broken), and  $45^\circ$  (dashdot). The fixed plasma parameters are  $R = 0.333$ ,  $T_c/T_h = 0.01$ , and  $n_{0c}/n_0 = 0.1$ .

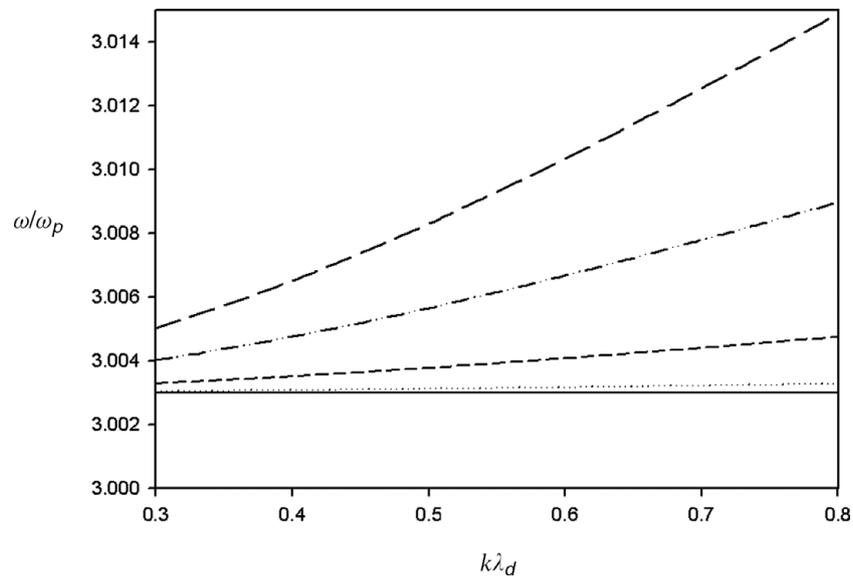
four-component, two-temperature electron–positron plasma and is due to the contribution of the second species. In a single-temperature electron–positron plasma, the electron-acoustic mode cannot exist. Figure 4 shows the variation of normalized frequency for various values of  $T_c/T_h$  ratios for the parameters of Fig. 3.

Figures 5 and 6 show the normalized real frequency versus normalized wavenumber for a range of propagation angles for electron-acoustic and cyclotron branches, respectively. It is noted that the slope of the curves are much smaller as compared to the single-temperature electron–positron model of Zank and Greaves (1995). Frequency of the electron-acoustic mode (Fig. 5) de-

creases with propagation angle and eventually disappears at  $\theta = 90^\circ$ . On the other hand, frequency of cyclotron mode increases with increase in propagation angle.

#### 4. Conclusions

Linear electrostatic waves in a magnetized four-component, two-temperature electron–positron plasma have been investigated, with the hot species having a Boltzmann density distribution and the dynamics of the cooler species governed by fluid equations. Solutions of the corresponding dispersion relation yield the electron-



**Figure 6.** Normalized real frequency as a function of the normalized wavenumber showing cyclotron branch for various angles of propagation  $\theta = 0^\circ$  (solid),  $9^\circ$  (dotted),  $22.5^\circ$  (broken),  $45^\circ$  (dashdotdot), and  $90^\circ$  (longdash). The other fixed plasma parameters are same as in Fig. 5.

acoustic, upper hybrid, electron plasma and electron cyclotron branches, which were explored as a function of several plasma parameters. For perpendicular wave propagation, an electron cyclotron mode exists with contributions from both cooler and hot species and hence influencing the dispersive properties of the wave. In the absence of hot species, this mode goes over to the upper hybrid mode where the cooler species contribute to the wave dynamics, as expected and reported earlier by Zank and Greaves (1995). On the other hand, for parallel propagation, the solutions display a dominant electron acoustic mode, which goes over to an electron plasma mode when hot species are absent. Further, this mode is decoupled from the electron cyclotron mode. The properties of other modes (cyclotron and upper hybrid) studied here are also significantly modified due to the presence of two-temperature (cold and hot) electron–positron populations.

The four-component magnetized plasmas consisting of cold and hot electrons–positrons can be present in pulsar magnetosphere. Such plasmas can support the electron-acoustic mode, a novel mode that is not present in the pure two-component, single-temperature electron–positron plasma. In the Earth’s magnetosphere electron-acoustic modes are important and could explain the broadband electrostatic noise (BEN) emissions up to the cold electron plasma frequency (Singh and Lakhina 2001). Although the BEN observations in the pulsar magnetosphere have not been reported, the electron-acoustic mode can play important role there. Firstly, it can modulate the pulsar electromagnetic radiation during their passage through the pulsar magnetosphere leading to modulational instability (Hasegawa 1975; Luo 1998). This would explain some features of microstructure in pulsar radiation. Secondly, the coupling of electron-acoustic wave with an electromagnetic

wave may produce, depending upon the beat conditions, higher or lower frequency electromagnetic waves by three-wave interaction process (Luo 1998).

In this paper we have carried out the analysis of linear electrostatic waves in four-component electron–positron plasmas using fluid equations. The results may be modified in the presence of kinetic effects. This is best studied using the kinetic theory, which we plan to do in future.

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