Satellite Look Angles, Track and Geometry in Mobile Satellite Communications

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Abstract: This paper describes Satellite Look Angles, Track and Geometry in the Space and their significance with regard to the spacecraft use for Mobile Satellite Communications (MSC) systems. Basic geometric theory of satellite coordinates is applied to determine the geographical azimuth and elevation (spacecraft altitude) angles required to point mobile satellite tracking antenna to the Geostationary Earth Orbit (GEO) or Non-GEO communication satellites. The mathematical treatment introduced in this paper takes into consideration circular and elliptical orbits. During more than three decades, commercial MSC networks have utilized GEO satellites extensively to the point where orbital portions have become crowded; coordination between satellites is becoming constrained and could never solve the problem of polar coverage. However, other Non-GEO MSC solutions have recently grown in importance because of their orbit characteristics and coverage capabilities in high latitudes and Polar Regions. The important parameters of the Satellite Look Angles such as Elevation and Azimuth angles and Satellite Track and Geometry such as Longitude and Latitude values are described.

Key Words: MSC, MES, LES, SES, GEO, Non-GEO, LEO, PEO, Elevation, Azimuth, Longitude, Latitude

1. Introduction

The satellite platform is located in orbit around the Earth at different altitudes, starting from 20 km in the stratosphere and up to 36,000 km in the Space. Orbital mechanics is a specific discipline describing planetary and satellite motions in the Solar system, which can solve the problems of calculating the look angles by the elevation and azimuth amounts, position by the longitude and latitude values, speed, path, perturbation and other orbital values of planets and satellites. Thus, these parameters are very important for the tracking satellite in focus of directional satellite antenna onboard Mobile Earth Station (MES).

2. Satellite Motion and Orbits

A satellite is an artificial object located by rocket in space orbit following the same laws in its motion as the planets rotating around the Sun. In this sense, three so important laws for planetary motion were derived by Johannes Kepler. At this point, Kepler's Laws were based on observational records and only described the planetary motion without attempting an additional theoretical or mathematical explanation of why the motion takes place in that manner. These conditions are not completely fulfilled in the case of Earth motion and its artificial satellites. However, in 1687, Sir Isaac Newton published his breakthrough work "Principia Mathematica" with own syntheses, known as the Three Laws of Motion **[1]**.

On the basis of Law II, Newton also formulated the Law of Universal Gravitation, which states that any two bodies attract one another with a force proportional to the products of their masses and inversely proportional to the square of the distance between them. This law may be stated mathematically for a circular orbit by relation:

$$F = m\,(2\pi/t)^2\,(R + h) = G\,[M\cdot m/(R + h)^2] \qquad (1)$$

where parameter m = mass of the satellite body in orbit; t = time of satellite orbit; R = equatorial radius of the Earth (6.37816×10^6 m); h = altitude of satellite platform above the Earth's surface; G = Universal gravitational constant (6.67×10^{-11} N m^2/kg^{-2}); M = Mass of the Earth body (5.976032×10^{24} kg) and finally, F = force of mass (m) due to mass (M).

The satellite in any circular orbit undergoes its revolution at a fixed altitude and with fixed velocity, while a satellite in an elliptical orbit can drastically vary its altitude and velocity during one revolution. The elliptical orbit is also subject to Kepler's Three Laws of satellite motion. In such a way, the characteristics of elliptical orbit can be determined from elements of the ellipse of the satellite plane with the perigee (Π) and apogee (A) and its position in relation to the Earth, see **Figure 1 (A).**

The parameters of elliptical orbit are presented as follows:

$$e = c/a = \sqrt{[1 - (b/a)^2]} \quad \text{or} \quad e = (\sqrt{a^2} - b^2/a)$$
$$p = a\,(1 - e^2) \quad \text{or} \quad p = b^2/a$$
$$c = \sqrt{(a^2 - b^2)} \qquad a = p/1 - e^2$$
$$b = a\,\sqrt{(1 - e^2)} \qquad (2)$$

where e = eccentricity, which determines the type of conical section; a = large semi-major axis of elliptical orbit; b = small semi-major axis of elliptical orbit; c = axis between centre of the Earth and centre of ellipse and p = focal parameter. The equation of ellipse derived from polar coordinates can be presented with the resulting trajectory equation as follows **[2, 3]**:

$$r = p/1 + e\cos\Theta \quad [m] \qquad (3)$$

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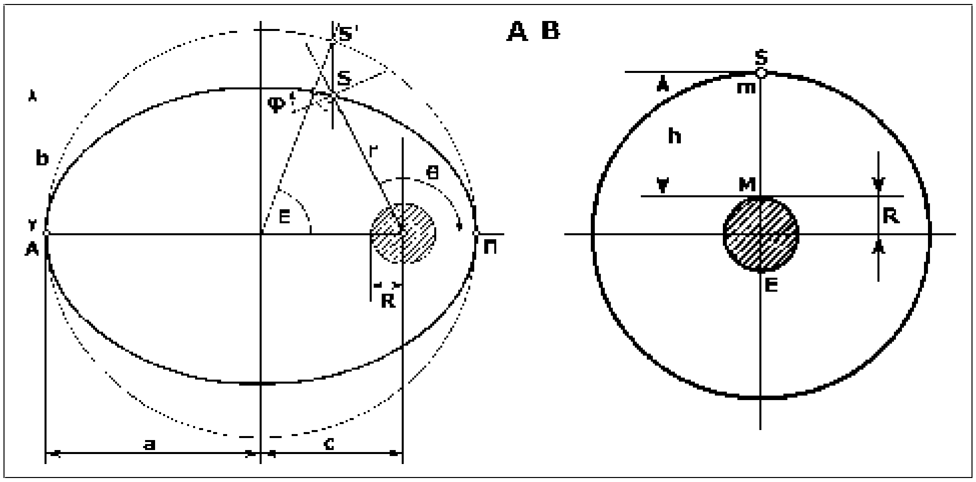

Figure 1. Elliptical and Circular Satellite Orbits – Courtesy of Book: "Telekomunikacije satelitima" by Galic **[3]**

where r = distance of the satellites from the centre of the Earth (r = R+h) or radius of path; Θ = true anomaly and E = eccentric anomaly. In this case, the position of the satellite will be determined by the angle called "the true anomaly", which can be counted positively in the direction of movement of the satellite from 0^o to 360^o, between the direction of the perigee and the direction of the satellite (S). The position of the satellite can also be defined by eccentric anomaly, which is the argument of the image in the mapping, and which transforms the elliptical satellite trajectory into its principal circle. Its angle is counted positively in the direction of movement of the satellite from 0 to 360^o, between the direction of the perigee and the direction of the satellite. The relations for both mentioned anomalies are given by the following equations:

$$\cos \Theta = \cos E - e/1 - e \cos E$$
$$\cos E = \cos \Theta + e/1 + e \cos \Theta \qquad (4)$$

The total mechanical energy of a satellite in elliptical orbit is constant; although there is an interchange between the potential and the kinetic energies. As a result, a satellite slows down when it moves up and gains speed as it loses height. Accordingly, considering the termed gravitation parameter μ=GM (Kepler's Constant $\mu = 3.99 \times 10^5$ km^3/sec^2), the velocity of a satellite in an elliptical orbit may be obtained from the following relation:

$$v = \sqrt{[GM (2/r) - (1/a)} = \sqrt{\mu} (2/r) - (1/a)] \qquad (5)$$

Applying Kepler's Third Law the sidereal time of one revolution of the satellite in elliptical orbit is as follows:

$$t = 2\pi \sqrt{(a^3/GM)} = 2\pi \sqrt{(a^3/\mu)}$$
$$t = 3.147099647 \sqrt{(26{,}628.16 \cdot 10^3)^3} \cdot 10^{-7} = 43{,}243.64 \quad [s] \qquad (6)$$

Therefore, the last equation is the calculated period of sidereal day for the elliptical orbit of Russian-based satellite Molnya with apogee = 40,000 km, perigee = 500 km, revolution time = 719 min and a = 0.5 (40,000 + 500 + 2 x 6,378.16) = 26,628.15 km.

The circular orbit is a special case of elliptical orbit, which is formed from the relations a = b = r and e = 0, shown in **Figure 1 (B).** According to Kepler's Third Law, the solar time (τ) in relation with the right ascension of an ascending node angle (Ω); the sidereal time (t) with the consideration that μ=GM and satellite altitude (h), for a satellite in circular orbit will have relations:

$$\tau = t /(1 - \Omega t / 2\pi) \qquad (7)$$

$$t = 2\pi \sqrt{(r^3/\mu)} = 3.147099647 \sqrt{(r^3 \cdot 10^{-7})} \quad [s] \qquad (8)$$

$$h = [\sqrt[3]{(\mu t^2/4\pi^2)}] - R = 2.1613562 \cdot 10^4 (\sqrt[3]{t^2}) - 6.37816 \cdot 10^6 \quad [m] \qquad (9)$$

The time is measured with reference to the Sun by solar and sidereal day. Thus, a solar day is defined as the time between the successive passages of the Sun over a local meridian. In fact, a solar day is a little bit longer than a sidereal day, because the Earth revolves by more than 360^o for successive passages of the Sun over a point 0.986^o further. On the other hand, a sidereal day is the time required for the Earth to rotate one circle of 360^o around its axis: t_E = 23 h 56 min 4.09 sec **[3, 4]**.

Therefore, any GEO communication satellite must have an orbital period of one sidereal day in order to appear stationary to an observer on Earth. During rotation the duration of sidereal day t = 85,164,091 (s) and is considered in such a way for synchronous orbit that h = $35{,}786.04 \times 10^3$ (m). The speed is conversely proportional to the radius of the path (R+h) and for the satellite in circular orbit it can be calculated from this relation:

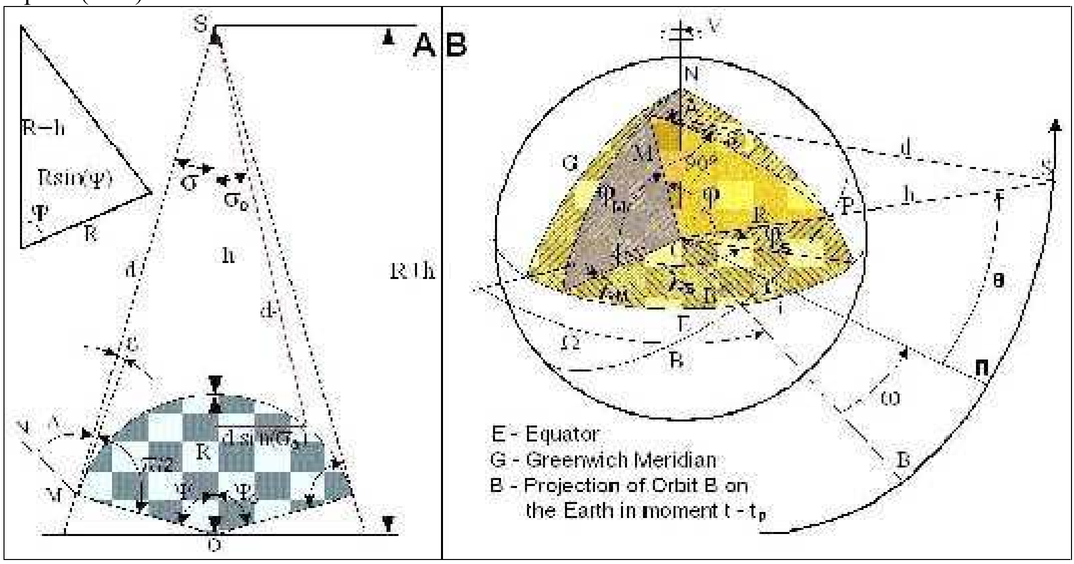

Figure 2. Geometric Projection of Satellite Orbits – Courtesy of Books: (A) "Sputnikovaya svyaz na more" by Novik **[5]** and (B) "Mezhdunarodnaya sputnikovaya sistema morskoy svyazi – Inmarsat" by Zhilin **[2]**

$$v = \sqrt{(MG/R + h)} = \sqrt{(\mu/r)} = 1.996502 \cdot 10^{-7}/\sqrt{r} = 631.65 \sqrt{r} \quad [m/s] \qquad 10)$$

From stated equation (8) and using the duration of sidereal day (t_E) gives the relation for the radius of synchronous or geostationary orbits:

$$r = {}^{3}\sqrt{[(\mu t) / 2\pi)^2]} \qquad (11)$$

The satellite trajectory can have any angle of orbital planes in relation to the equatorial plane: in the range from Polar Earth Orbit (PEO) up to GEO plane. Thus, if the satellite is rotating in the same direction of Earth's motion, where (t_E) is the period of the Earth's orbit, the apparent orbiting time (t_a) is calculated by the following relation:

$$t_a = t_E \cdot t/t_E - t \qquad (12)$$

This means, inasmuch as $t = t_E$ the satellite is GEO (geostationary), where $t_a = \infty$ or τ=0. **Table 1** shows several values for different times.

Table 1. The Values of Times Different than the Synchronous Time of Orbit.

Parameter	Values of Time					Unit
t	86,164.00	43,082.05	21,541.23	10,770.61	6,052.00	s
h	35,786.00	20,183.62	10,354.71	4,162.89	800.00	km
(R+h)	42,164.00	26,561.78	16,732.87	10,541.05	7,178.00	km
v	3,075.00	3,873.83	4,880.72	5,584.12	7,450.00	km/s^{-1}

According to **Table 1** and stated equation (9) it is evident that a satellite does not depend so much on its mass but decreases with higher altitude. In addition, satellites in circular orbits with altitudes of a 1,700, 10,400 and 36,000 km, will have t/ τ values 2/2,18, 6/8 and 24/zero, respectively. In this case, it is evident that only a satellite constellation at altitudes of about 36,000 km can be synchronous or geostationary **[2, 5, 6]**.

3. Horizon and Geographic Satellite Coordinates

The geographical and horizon coordinates for each satellite are important to determine satellite parameters and equations for better understanding the problems of satellite orbital plane, satellite distance, visibility of the satellite, coverage areas, etc. The coverage areas of a satellite are shown in **Figure 2 (A)** and geometrical parameters: actual altitude (h), radius of Earth (R), angle of elevation (ε), angle of azimuth (A), distance between satellite and the Earth's surface (d) and central angle (Ψ) or sub- satellite angle, which is similar to the angle of antenna radiation (δ).

The satellite geographical and horizon coordinates are shown in **Figure 2 (B)** including parameters: angular speed of the Earth's rotation (ν), argument of the perigee (ω), moment of satellite pass across any point on the orbit (t_o), which can be perigee (Π), projection of the perigee point on the Earth's surface (Π'), spherical triangle (B'ΓP), satellite (S), observer or mobile point (M), latitudes of (φ_M and φ_S) and longitudes of observer and satellite (λ_M and λ_S), inclination angle (i) of the orbital plane measured between the equatorial and orbital plane and the right ascension of an ascending Otherwise, the right ascension of an ascending node angle (Ω) is the angle in the equatorial plane measured counter clockwise from the direction of the vernal equinox to that of the ascending node, while the argument of the perigee (ω) is the angle between the direction of the ascending node and the direction of the perigee **[2, 5, 7]**.

3.1. Satellite Distance and Coverage Area

The area coverage or angle of view for each type of satellite depends on orbital parameters, its position in relation to the Land Earth Station (LES) or observer and geographic coordinates. This relation is very simple in the case where the sub-satellite point is in the centre of coverage, while all other samples are more complicated. In such a way, the angle of a GEO satellite inside its range will have the following regular reciprocal relation:

$$\delta + \varepsilon + \Psi = 90^o \quad (13)$$

The circular sector radius can be determined by the following relation:

$$R_s = R \sin \Psi \quad (14)$$

When the altitude of orbit (h) is the distance between satellite and sub-satellite point (SP), the relation for the altitude of the circular sector can be written as:

$$h_s = R\,(1 - \cos \Psi) \quad (15)$$

From a satellite communications point of view, there are three key parameters associated with an orbiting satellite: **(1)** Coverage area or the portion of the Earth's surface that can receive the satellite's transmissions with an elevation angle larger than a prescribed minimum angle; **(2)** The slant range (actual line-of-sight distance from a fixed point on the Earth to the satellite) and **(3)** The length of time a satellite is visible with a prescribed elevation angle. Elevation angle is an important parameter, since communications can be significantly impaired if the satellite has to be viewed at a low elevation angle, that is, an angle too close to the horizon line. In this case, a satellite close to synchronous orbit covers about 40% of the Earth's surface.
Thus, from the diagram in **Figure 2 (A)** a covered area expressed with central angle (2δ or 2Ψ) or with arc ($MP \approx R\Psi$) as a part of Earth's surface that can be derived with the following relation:

$$C = \pi\,(R_s^2 + h_s^2) = 2\pi R^2\,(1 - \cos \Psi) \quad (16)$$

node angle in the moment of t_o (Ω_o).

Since the Earth's total surface area is $4\pi R^2$, it is easy to rewrite (C) as a fraction of the Earth's total surface:

$$C/4\pi R^2 = 0{,}5\,(1 - \cos \Psi) \quad (17)$$

The slant range between a point on Earth and a satellite at altitude (h) and elevation angle can be defined in this way:

$$z = [(R \sin \varepsilon)^2 + 2Rh + h^2]^{½} - R \sin \varepsilon \quad (18)$$

This determines the direct propagation length between LES, (h) and (ε) and will also find the total propagation power loss from observer or LES to satellite. In addition, (z) establishes the propagation time (time delay) over the path, which will take an electromagnetic field as:

$$t_d = (3.33)\ z \quad [\mu sec] \quad (19)$$

To propagate over a path of length (z) km, it takes about 100 msec to transmit to GEO. If the satellite location is uncertain ± 40 km, a time delay of about ± 133 μsec is always presented in the Earth-to-satellite propagation path. When satellite is in orbit at altitude (h), it will pass over a point on Earth with elevation angle (ε) for the time period:

$$t_p = (2\Psi/360)\ (t/1 \pm (t/t_E) \quad (20)$$

In such a way, the quotations for right ascension of the ascending node angle (Ω) and argument of the perigee (ω) are as follows:

$$\Omega = 9{,}95\ (R/r)^{3.5} \cos i \quad \text{or} \quad \Omega = \Omega_o + \nu\ (t - t_o)$$
$$\omega = 4{,}97\ (R/a)^{3.5}\ [5 \cos^2 i - 1/(1 - e^2)^2] \quad (21)$$

The limit of the coverage area is defined by the elevation angle from LES above the horizon with angle of view $\varepsilon = 0^o$. Thus, the satellite is visible and its maximal central angle for GEO will be:

$$\Psi = \text{arc}\cos\,(R \cos \varepsilon/r) - \varepsilon \quad \text{or}$$
$$\Psi = \pi/2 - \text{arc}\sin\,(R/r) = \text{arc}\cos\,(R/r) - \varepsilon = \text{arc}\cos k - \varepsilon$$
$$\Psi = \text{arc}\cos 6{,}376.16/42{,}164.20 = \text{arc}\cos 0.15126956 = 81^o\ 17'\ 58.18''$$
$$C_{max} = 255.61 \cdot 10^6\ (1 - 0.15126956) = 216.94 \cdot 10^6\ (km^2) \quad (22)$$

where k = Boltzmann's constant, 1.38× 0–23 J/K.
All MES and LES with a position above $\Psi = 81^o$ will be not covered by GEO satellites. Since the Earth's square area is 510,100,933.5 km^2 and the extent of the equator is 40,076.6 km, only with

three GEO apart in the orbit by 120° it is possible to cover entire Earth's surface **[3, 5, 8].**

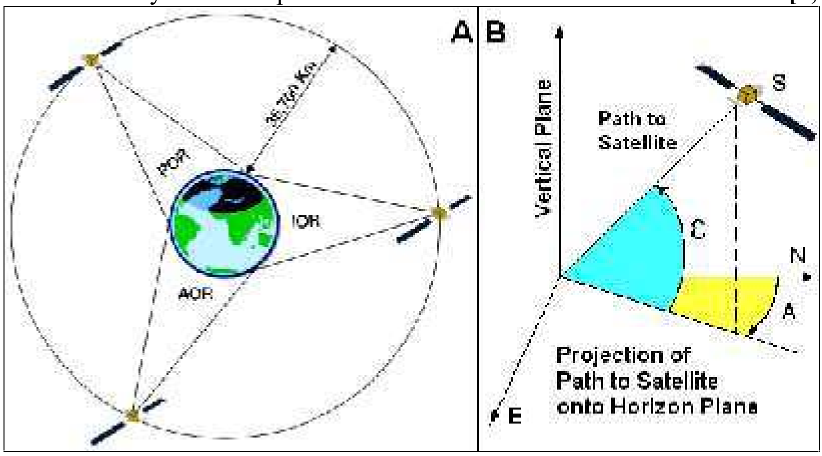

Figure 3. GEO Satellite Configuration and Look Angle Parameters – (A) "Telekomunikacije satelitima" by Galic **[3]** and (B) "Satellite Communications" by Pratt **[8]**

Therefore, three satellites cover 3 Ocean Regions: Atlantic AOR), Indian (IOR) and Pacific (POR), which is shown in **Figure 3 (A)**. The zero angles of elevation have to be avoided, even to get maximum coverage, because this increases the noise temperature of the receiving antenna. Owing to this problem, an equation for the central angle with minimum angle of view between 5° and 30° will be calculated with the following relation:

$$\Psi_s = \text{arc cos } (k \cos \varepsilon) - \varepsilon \qquad (23)$$

The arch length or the maximum distant point in the area of coverage can be determined by:

$$l = 2\pi R\ (2\Psi/360 = 222.64\Psi \quad [\text{km}] \qquad (24)$$

The real altitude of satellite over sub-satellite point is as follows:

$$h = r - R = 42{,}162 - 6{,}378 = 35{,}784 \quad [\text{km}] \qquad (25)$$

The view angle under which a GEO satellite can see LES/MES is called the "sub-satellite angle". More distant points in the coverage area for GEO satellites are limited around φ=70° of North and South geographical latitudes and around λ=70° of East and West geographical longitudes, viewed from the sub-satellite's point. Theoretically, all Earth stations around these positions are able to see satellites by a minimum angle of elevation of ε=5°. Such access is very easy to calculate, using simple trigonometry relations **[3, 9]**:

$$\delta_{\varepsilon=0} = \text{arc sin } k \approx 9^o \qquad (26)$$

At any rate, the angle (Ψ) is in correlation with angle (δ), which can determine the aperture radiation beam. For example, the aperture radiation beam of satellite antenna for global coverage has a radiation beam of 2δ=17.3°.
According to **Figure 2 (A)** it will be easy to find out relations for GEO satellites as follows:

$$\text{tg } \delta = k \sin \Psi/1 - k \cos \Psi = 0.15126956 \sin \Psi/1 - \cos \Psi/1 - 0.15126956 \cos \Psi)$$
$$\delta_s = 90^o - \Psi_s = 8^o\, 42'\ 1.82'' \qquad (27)$$

The width of the beam aperture (2δ_s) is providing the maximum possible coverage for synchronous circular orbit. The distance of LES and MES with regard to the satellite can be calculated using **Figure 2 (A)** and equations (13) and (22) by:

$$d = R \sin \Psi/\sin \delta = r \sin /\cos \varepsilon \qquad (28)$$

Parameter (d) is important for transmitter power regulation of LES, which can be calculated by the following equation:

$$d = \sqrt{[}(R + r)^2 - 2R\, r \cos \Psi] \quad \text{or}$$
$$d = h \sqrt{[}1 + 2\ (1/k)\ (R/h)^2\ (1 - \cos \varphi \cos \Delta\lambda)] \quad \text{or}$$
$$d = r\ [1 - (R \cos \varepsilon/r)^2]^{1/2} - R \sin \varepsilon \qquad (29)$$

Accordingly, when the position of any MES is near the equator in sub-satellite point (P) or right under the GEO satellite, then its distance is equal to the satellite altitude and takes out value for d=H of 35,786 km. Thus, every MES will have a further position from (P) when the central angle exceeds Ψ = 81°, when d_{max}=41,643 km **[5, 10]**.

3.2. Satellite Look Angles (Elevation and Azimuth)

The horizon celestial coordinates are considered to determine satellite position in correlation with an Earth observer, LES and MES terminals. These specific and important horizon coordinates are angles of satellite elevation and azimuth values, presented in **Figure 2 (A** and **B)** and **Figure 3 (B),** respectively **[2, 5, 8].**

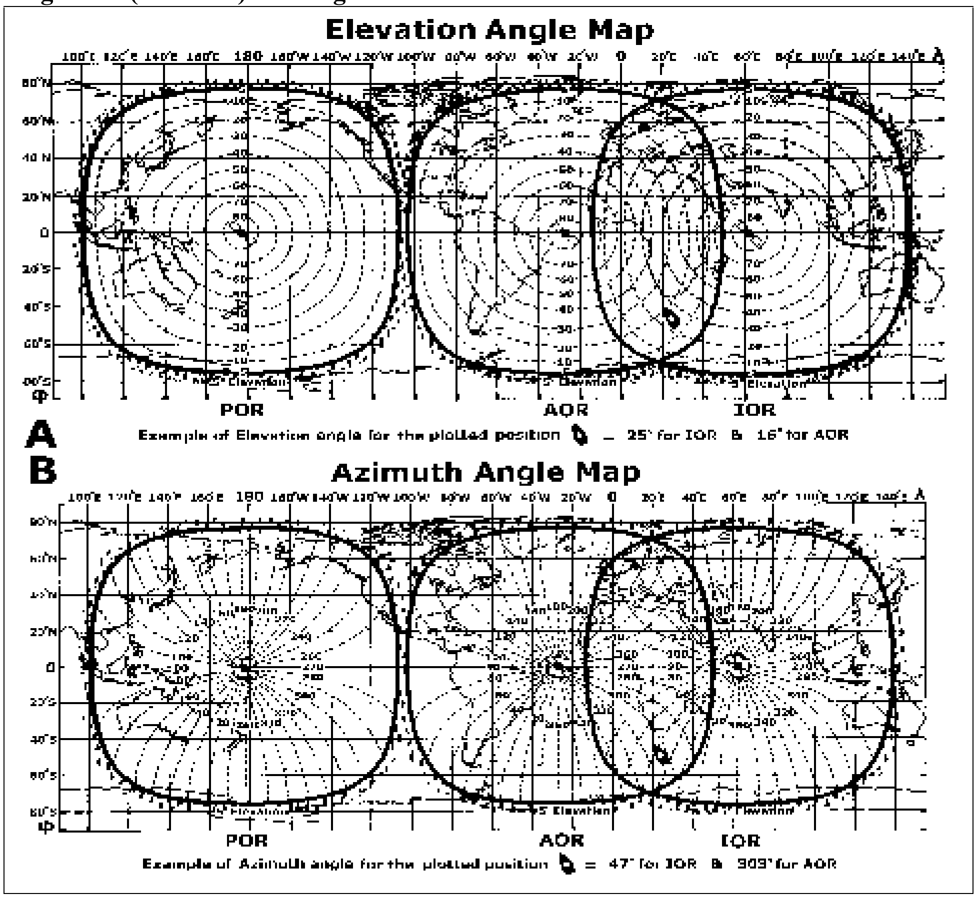

Figure 4. Elevation and Azimuth Angle Maps – Courtesy of Manual: "Saturn 3 MES" by EB **[11]**

The satellite elevation (ε) is the angle composed upward from the horizon to the vertical satellite direction on the vertical plane at the observer point. From point (M) shown in **Figure 2 (A)** the look angle of (ε) value can be calculated by the following relation:

$$\text{tg}\ \varepsilon = \cos \Psi - k/\sin \Psi \qquad (30)$$

In **Figure 4 (A)** is illustrated the Mercator chart of the 1st Generation Inmarsat space segment, using three ocean coverage areas with projection of elevation angles and with one example of a plotted position of a hypothetical ship (may also be aircraft or any mobile).

Therefore, it can be concluded that Ship Earth Station (SES) or any type of MES at designated position (ε=25° for IOR and ε=16° for AOR) has the possibility to use either GEO satellites over IOR or AOR to communicate with any LES inside the coverage areas of both satellites.

The satellite azimuth (A) is the angle measured eastward from the geographical North line to the projection of the satellite path on the horizontal plane at the observer point. This angle varies between 0 and 360° as a function of the relative positions of the satellite and the point considered.

The azimuth value of the satellite and sub-satellite point looking from the point (M) or the hypothetical position of MES can be calculated:

$$\text{tg}\ A' = \text{tg}\ \Delta\lambda_M - k/\sin \Psi \qquad (31)$$

Otherwise, the azimuth value, looking from sub-satellite point (P), can be calculated as:

$$\text{tg}\ A = \sin \Delta\lambda / \text{tg}\ \varphi \quad \text{or}$$
$$\sin A = \cos \varphi \sin \Delta\lambda\ \text{cosec}\ \Psi \qquad (32)$$

In **Figure 4 (B)** is shown the Mercator chart of 1st Generation Inmarsat 3-satellite or ocean coverage areas with projection of azimuth angles sample for the plotted position of a hypothetical ship (ε=47° for IOR and ε=303° for AOR area). Any mobile inside of both satellites' coverage can establish a link to the subscribers on shore via any LES.

Parameter (A') is the angle between the meridian plane of point (M) and the plane of a big circle crossing this point and sub-satellite point (P), while the parameter (A) is the angle between a big circle and the meridian plane of point (P). The elevation and azimuth are respectively vertical or horizontal look angles, or angles of view, in which range the satellite can be seen **[4, 11]**.

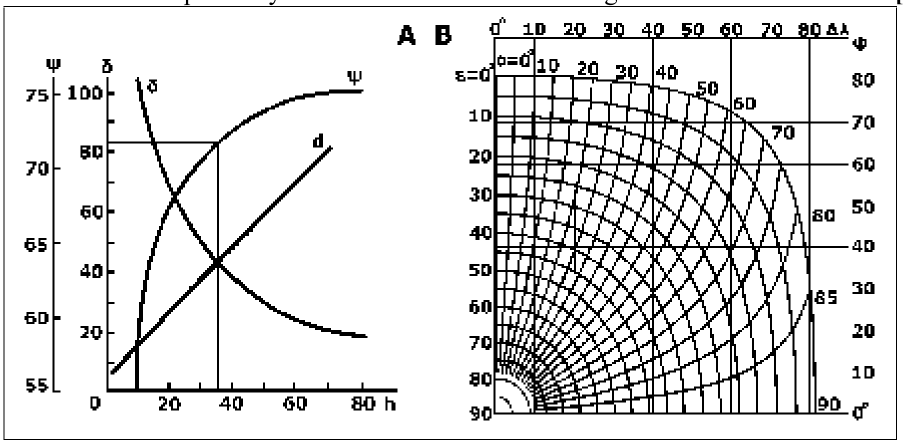

Figure 5. Look Angle Parameters and Graphic of Geometric Coordinates for GEO – Courtesy of Book: "Mezhdunarodnaya sputnikovaya sistema morskoy svyazi – Inmarsat" by Zhilin **[2]**

In **Figure 5 (A)** is presented a correlation of the look angle for three basic parameters (δ, Ψ, d) in relation to the altitude of the satellite. Inasmuch as the altitude of the satellite is increasing as the values of central angle (Ψ), distance between satellite and the Earth's surface (d) and duration of communication (t_c) or time length of signals are increasing, while the value of sub-satellite angle (δ) is indirectly proportional. At this point, an important increase of look angle and duration of communication can be realized by increasing the altitude to 30 or 35,000 km, while an increase in look angle is unimportant for altitudes of more than 50,000 km. The duration of communication is affected by the direction's displacement from the centre of look angle, which will have maximum value in the case when the direction is passing across the zenith of the LES. The single angle of the satellite in circular orbit depends on the t/2 value, which in area of satellite look angle, can be found in the duration of the time and is determined as:

$$t_c = \Psi\, t / \pi \qquad (33)$$

However, practical determination of the geometric parameters of a satellite is possible by using many kinds of plans, graphs and tables. It is possible to use tables for positions of SES (φ, λ), by the aid of which longitudinal differences can be determined between MES and satellite for four feasible ship's positions: N/W, S/W, N/E and S/E in relation to GEO spacecraft.

One of the most important practical pieces of information about a communications satellite is whether it can be seen from a particular location on the Earth's surface. In **Figure 5 (B)** a graphic design is illustrated, which can approximately determine limited zones of satellite visibility from the Earth (MES) by using elevation and azimuth angles under the condition that δ = 0. This graphic contains two groups of crossing curves, which are used to compare (φ) and (Δλ,) coordinates of mobile positions. Thus, the first group of parallel concentric curves shows the geometric positions where elevation has the constant value (ε=0), while the second group of fan-shaped curves starting from the centre shows the geometric positions where the difference in azimuth has the constant value (a = 0). For example, this diagram can be used in accordance with **Figure 2 (B)** in the following orders:

1) First, for different mobile determinations, it is necessary to note the longitude values of satellite (λ_S) and mobile (λ_M) and the latitude of the mobile (φ_M), then calculate the difference in longitude (Δλ) and plot the point into the graphic with both coordinates (φ_M & Δλ).

2) The value of elevation angle (ε) can then be determined by a plotted point from the group of parallel concentric curves.

3) The difference values of azimuth (a) angle can be determined by a plotted point from the group by fan-shaped curves starting from the centre of the graphic.

4) Finally, depending on the mobile position, the value of azimuth (A) can be determined on the basis of the relations presented in **Table 2.**

Table 2. The Form for Calculation of Azimuth Values

The GEO direction in relation to MES	Calculating of Azimuth Angles
Course of MES towards S & W	A = a
Course of MES towards N & W	A = 180° – a
Course of MES towards N & E	A = 180° + a

Course of MES towards S & E	$A = 360^o - a$

Inasmuch as the position of SES is of significant or greater height above sea level (if the bridge or ship's antenna is in a very high position) or according to the flight altitude of Aircraft Earth Station (AES), then the elevation angle will be compensated by the following parameter:

$$x = \arccos (1 - H/R) \quad (34)$$

where H = height above sea level of observer or MES. If the position of LES is a height of H = 1,000 m above sea level, the value of $x \approx 1^o$. This example can be used for the determination of AES compensation parameters, depending on actual aircraft altitude. Thus, the estimated value of elevation angle has to be subtracted for the value of the compensation parameter (x) **[2, 3, 4]**.

3.3. Satellite Track and Geometry (Longitude and Latitude)

The satellite track on the Earth's surface and the presentation of a satellite's position in correlation to the MES results from a spherical coordinate system, whose centre is the middle of Earth, illustrated in **Figure 2 (B)**. The satellite position in any time can be decided by the geographic coordinates, sub-satellite point and range of radius. The sub-satellite point is a determined position on the Earth's surface; above it is the satellite at its zenith.

Thus, the longitude and latitude are geographic coordinates of the sub-satellite point, which can be calculated from the spherical triangle (B'ΓP), using the following relations:

$$\sin \varphi = \sin (\Theta + \omega) \sin i$$
$$\mathrm{tg}\, (\lambda_S - \Omega) = \mathrm{tg}\, (\Theta + \omega) \cos i \quad (35)$$

With the presented equation in previous relation it is possible to calculate the satellite path or trajectory of sub-satellite points on the Earth's surface. The GEO track breaks out at the point of coordinates $\varphi = 0$ and λ = const.

Furthermore, considering geographic latitude (φ_M) and longitude (λ_M) of the point (M) on the Earth's surface is presented in **Figure 2 (B).** This can be the position of the MES, however taking into consideration the arc (MP) of the angle presented in **Figure 2 (A)**, the central satellite angle can be calculated by the following relations:

$$\cos \Psi = \cos \varphi_S \cos \Delta\lambda \cos \varphi_M + \sin \varphi_S \sin \varphi_M \quad \text{or}$$
$$\cos \Psi = \cos \text{arc MP} = \cos \varphi_M \cos \Delta\lambda \quad (36)$$

The transition calculation from geographic to spherical coordinates and vice versa can be computed with the following equations:

$\cos \Psi = \cos \varphi \cos \Delta\lambda$ and $\mathrm{tg}\, A = \sin \Delta\lambda / \mathrm{tg}\, \varphi$, respectively

$\sin \varphi = \sin \Psi \cos A$ and $\mathrm{tg}\, \Delta\lambda = \mathrm{tg}\, \Psi \sin A$ (37)

These relations are useful for any point or area of coverage on the Earth's surface, then for a centre of the area if it exists, as well as for spot-beam and global area coverage for MSC systems. The optimum number of GEO satellites for global coverage can be determined by:

$$n = 180^o/\Psi \quad (38)$$

For instance, if $\delta = 0$ and $\Psi = 81^o$, it will be necessary to put into orbit only 3 GEO, and to get a global coverage from 70^o N to 70^o S geographic latitude. Hence, in a similar way the number of satellites can be calculated for other types of satellite orbits. The trajectory of radio waves on a link between an MES and satellite at distance (d) and the velocity of light ($c = 3 \times 10^8$ m/s) require a propagation time equal to:

$$T = d/c \quad (s) \quad (39)$$

Phenomenon of apparent change in frequency of signal waves at the receiver when the signal source moves with respect to the receivers (Earth), was explained and quantified by Johann Doppler (1803–53). The frequency of the satellite transmission received on the ground increases as the satellite is approaching the ground observer and reduces as the satellite is moving away. This change in frequency is called Doppler Effect or shift, which occurs on both the uplink and the downlink. It is quite pronounced for Low Earth Orbits (LEO) and compensating for it requires frequency tracking in a narrowband receiver, while its effect are negligible for GEO satellites. The Doppler shift at a transmitting frequency (f) and radial velocity (v_r) between the observer and the transmitter can be calculated by relation:

$$\Delta f_D = f\, v_r / c \quad \text{where} \quad v_r = dR/dt \quad (40)$$

For an elliptical orbit, assuming that R = r, the radial velocity is given by:

$$v_r = dr/dt = (dr/\Theta)\,(d\Theta/dt) \quad (41)$$

The sign of the Doppler shift is positive when the satellite is approaching the observer and vice versa. Doppler Effect can also be used to estimate the position of an observer provided that the orbital parameters of the satellite are precisely known. In such a mode, this is very important for development of Doppler satellite tracking and determination systems **[2, 5, 6].**

4. Conclusion

All type of satellites are well suited for all mobile applications because of their capability to enhance coverage and support long-range mobility using mobile satellite devices and satellite antenna. Mostly of satellite transceivers onboard mobiles need suitable satellite tracking antennas, which tracking mechanism has to point antenna to the certain satellite. At this sense, the orbital coordinates, such as elevation and azimuth look angles including geographical values of longitude and latitude are essential for precise pointing of satellite mobile antenna in focus of adequate satellite.
Satellites provide the best and very attractive alternative for commercial, military and distress communications and navigation, including mobile DVB-RCS solutions and access to the Internet. In fact, satellites are attractive in sparsely populated areas, where high bandwidth of radio cellular systems cannot be economically deployed or in impervious regions where deployment of any terrestrial facilities is not practical. Today several MSC operators provide global, regional and local coverage for all applications via both GEO and Non-GEO or LEO satellites.

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