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The Philosophical Dimensions of Mathematics in Engineering Education

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Abstract

Philosophical dimensions of mathematics education have drawn the attention of engineering educators and professionals in the 21st century, as it is perceived as a significant aspect in engineering education. Thus, engineering education has not witnessed critical changes with respect to mathematics as a lot of concerns such as staff capacity building and development, poor funding, and obsolete curriculum has become impediments in understanding the conceptual interpretation of mathematical philosophy in engineering education, hence the crux of this paper. This paper was guided by Social Constructivism Theory, which looks at how mathematics speciality forms its own epistemic knowledge-based domain that enhances and improves engineering education and contributes immensely to building potential engineering scholars. This paper addresses problems by filling the gaps in motivating the core investigation of unifying conjectures that are related to different areas of mathematics. This paper takes a broad look at the philosophical dimensions of mathematics in engineering education, while examining how mathematics as a body of knowledge is accounted as a means of enquiry in engineering education. The paper assesses the social philosophy and ethical dimensions of mathematics and its associated benefits; as well as its implications for engineering education (EE) in the 21st century. Thus, to address these gaps, recommendations on restructuring engineering education in areas such as curriculum revision, faculty capacity building and development as well as university funding in equipping students with contemporary mathematics textbooks were suggested.

Keywords— Dimensions, engineering education, mathematics, philosophy, social constructivism, 21st century

I. INTRODUCTION

Worldwide, educational systems ranging from primary to higher education learning institutions have a constant theme that is likened to a universal vocalized language called mathematics. The simple description of mathematics is centred on its definition depending on the geographical location of the educational systems. In this paper, mathematics is described as an education in numeric sciences, that uses a range of different approaches including algebra, calculus, and basic arithmetic (Ernest, 2018; Li et al., 2019a; 2019b). While mathematics is a key element of various disciplines varying from sciences to social sciences, education, technology and among others; it is widely recognized to focus on the understanding and testing theories in mathematical and scientific discussion that is sometimes called 'pure mathematics'. Besides, core and new engineering disciplines requires mathematics as a primary subject that is taken as a compulsory course in engineering education globally. Mathematics has always been a hard subject to study hence, a lot of students have been seen to be struggling with a 'fear factor' associated with it. In engineering, there are several areas of professions that reflects that each branch has different study that is set to focus on the importance of mathematics (Zhao et al., 2016; Li et al., 2019).

The role of mathematics in engineering is to solve equations involving mathematical concepts and building sound knowledge-based learning integrals of mathematics in engineering education. This is seen as an inter-connection with engineering concepts of theoretical principles and theorems, which makes engineering students to have an advantage over mathematics as they develop a capability to grasp the basics of advanced difficult concepts in core engineering subjects (Borba et al., 2016; Marshall et al., 2017; Ernest, 2018). Thus, in engineering, students have to learn all about extensive use of Laplace and Fourier transforms equations, relating to linear algebra in circuit theory and signal processes. This becomes imperative as complex mathematical algebra finds maximum use in solutions of sinusoidal excited circuits equations and power system load flow problems (Ernest, 2018; Wedding et al., 2018; Trenholm et al., 2019). It is required to have a broad understanding of the typical concept that expands knowledge and understanding of the different mathematical concepts based on differential, integral calculus, and complex numbers. Significantly, mathematical engineering works exclusively with differential equations that are used on a daily basis in engineering profession (Borko et al., 2017; Blotnicky et al., 2018).

Engineering equations and concepts share a common similarity with mathematical equations where the angles of the structure are studied to ascertain structural functionality. The mathematical dimensions in engineering applications are often centered on algebra, calculus, and trigonometry; that involve basic principles of accounts and statistics. Mathematics has a vital role to play in many engineering courses as a good mathematical tool can be a significant asset to a successful engineering project (Jacobs et al., 2017; Jehopio et al., 2017). Thus, teaching mathematics in engineering education is very crucial, as new undergraduate students should be aware that they need mathematical concepts in professional activities. This paper presents a systematic review methodology by exploring the philosophical dimensions of mathematics in engineering education. The main objective of this paper is to fill the research gap by contributing to the overall understanding of the philosophical dimensions of mathematics in engineering education. Specifically, we explore how mathematics as a body of knowledge is accounted as a means of enquiry in mathematics education; and to assess the social philosophy and ethical dimensions of mathematics education and its associated benefits; as well as its implications for EE in the 21st century, hence, recommendations were suggested.

II. METHODOLOGICAL APPROACH

This paper adopted the Systematic Review Methodology, which is a logical and appropriate step, allowing the findings of separate reviews to be compared and contrasted, hence, providing engineering faculty members with the findings they need. Large number of studies on the philosophical dimensions of mathematics in engineering education have been published by engineering educators and professionals in the 21st century (Mallett et al., 2012; Ouhbi et al., 2015). This research systematically reviews these different authors based on research objectives, assess, review and convey existing studies in a single space. The methods used to identify and evaluate published reviews systematically, are drawn from existing studies (Leandro Cruz et al., 2019; Trevelyan, 2019), following scientific research practices in the conduct and reporting of systematic reviews are explicitly explained.

Systematic review approach identifies and appraises published articles from year 1951 to 2019 in the fields of Engineering and mathematics education systematically. The published articles were divided into three groups: the first was themed as core articles on the mathematical philosophy (year 1951 to 1980s), the second aspect was collated based on the application of mathematical philosophy in engineering education (year 1990 to 2000), while the third aspect was centred on the dimensions of mathematics education in engineering education (year 2001 to 2019). The purposes of this methodology are to evaluate published reviews of the core interpretations of philosophy in mathematics education; to enhance the significance and value of engineering education; to describe and discuss its implications in order to provide the best recommendations to engineering faculty, professionals, and relevant stakeholders. Thus, the process of identifying and appraising all published reviews allows researchers to describe the quality of the compiled existing studies, summarize and compare the conclusions of the reviews as well as discuss the implications and recommendations of the conclusions of the reviews (Bornasal et al., 2018; Wilson-Lopez et al., 2020).

III. OVERALL LITERATURE REVIEW STEM

The philosophical dimensions in mathematical education in engineering disciplines has recently brought together diverse recent and significant developments that explores the philosophy of mathematics in engineering education. The intervals in philosophical dimensions in engineering educational mathematics has prepared and offered a balance between philosophy of engineering education and philosophy in mathematics education *(Sun et al., 2012; Treholm et al., 2019).* A lot of debates have been raised to draw a widespread attention of the implementation and evaluation of mathematical philosophy in engineering educational curriculum to have a wider understanding of philosophy dimensions in mathematics education *(Li et al., 2014; Nye et al., 2018)*. The philosophy of mathematics in engineering education becomes an opportunity to many scientific and technological fields as it has continuously increased quantitative and computational literacy demands in the 21st century *(Li, 2018a; 2018b)*.

Yet changes in practices of how mathematical philosophy is viewed and taught in engineering education has revealed numerous shortcomings of the philosophical dimensions in mathematics education. Thus, it is not surprising that vital progress has not been made in this aspect, as mathematics is usually conceptualized to present a knowledge body of content to be learned. Instead, mathematics studied can be viewed as the methodization of skills made through various practices involving problem solving, reasoning, communicating, and mathematical modeling *(Harris et al., 2014; Liljedah et al., 2016; Ernest, 2018)*. Thus, much of the inductive part of mathematics has been lost, and the deductive part is often presented as rote procedures rather than a form of sense making of how formal mathematics can serve to organize and systematize mathematical models. Hence, the historical background of the mathematical philosophy is key in discussing different views in regarding the nature of mathematics in engineering education in order to problematize traditional approaches to mathematics teaching and learning (Freeman et al., 2014; Marshall et al., 2017). In contrast to the primarily utilitarian approaches that preceded them, the Greeks pioneered the study of mathematics for its own sake and pursued the development and the use of generalized mathematical theories and proofs, especially in geometry and measurement of mathematical theoretical models. Different perspectives about the nature of mathematics were gradually developed during that era (Ernest, 1997; 1998; 2018). For instance, Plato perceived the study of mathematics as pursuing the truth that exists in external world beyond individual's mind.

Since 1900s, there has been a gradual development of school mathematics, as conception of the nature of mathematics has increasingly received attention from mathematics educators (Ernest, 2018; Trenholm et al., 2019). The conception of mathematics in engineering education adopts and uses a direct and strong impact in the manner in which school mathematics should be presented and approached (Borba et al., 2016; Li et al., 2019). Similarly, the 'New Math' movement of the 1950s and 1960s used the formalism school of thought as the core to reform efforts as the content was presented in a structural format, using the set theoretical language and conceptions. But the outcomes were not a successful progression towards school mathematics that is best for students and teachers to adopt. Alternatively, the review of the nature of mathematics by Dossey (1992), identified and selected scholars' work and ideas applicable to both professional mathematicians and mathematics educators (Dossey, 1992).

Moreover, Delvin (2000) argued that mathematics is not a single entity but has four different faces such as: computation, formal reasoning, and problem solving; a way of knowing; a creative medium; and applications. Further, Delvin (2000) contended that school mathematics typically focuses on the first face, makes some reference to the fourth face, but pays almost no attention to the other two faces. The conception of mathematics was made to assemble ideas from the history of mathematics and observes mathematical activities that occurs across different settings (Ernest, 2018; Sun et al., 2018). Thus, the nature of mathematics can be understood as having different faces, rather than being governed by any single school of thoughts. At the same time, the ideas of Plato and Aristotle continue to influence the ways that mathematicians, mathematics educators, and the general public perceive mathematics (Delvin, 2012; Hayward et al., 2018). The evolving conceptions about the nature of mathematics in history suggests that there is room for individuals to decide how mathematics can be perceived, rather than being bounded by a pre-occupied notion of mathematics as 'given' or 'fixed'. Hence, each and every learner can experience mathematics through different practices and 'own' mathematics as a human activity.

Exploring mathematics as a body of knowledge accounted as a means of inquiry in mathematics education

In the past decades, numerous developments in the historical antecedents of the dimensions of the philosophy of mathematics has emerged as an important background in the philosophy of mathematics (Reichenbach, 1951; Durkin et al., 1991; Ernest, 2018). To some extent, the status of the philosophy of mathematics is determined, relative to its context and dependence on historical contingency. Though, a growing number of scholars have questioned the universality, absoluteness and perfectibility of mathematics and mathematical knowledge, pointing towards the controversies in mathematical and philosophical circles, although less so in education and in the social and human sciences (Richards, 1991; Livingstone, 1986). A major concern of this controversy was a re-examination of the role and purpose of proof in mathematics, which clearly can serve to permit mathematical claims and theorems that can no longer be taken as the prearrangement of the aim and determined array of demonstrated unconditional logical cogency. Hence, mathematical theorems and proofs may be likened to fulfil a mixture of functions, which illustrates the bonds between pedagogical aspects help in working mathematicians to develop and to extend methodological knowledge in demonstrating the existence of mathematical ontological (Fuller, 1993; Knuth, 1985). This becomes imperative as it can persuade mathematicians to validate the epistemological claims in mathematics education and as well as in engineering education.

Mathematics as a body of knowledge can be justified as a means of inquiry in mathematics education; new forms of mathematical knowledge are created as an important component for mathematics education, including tacit knowledge, knowledge of particulars, language, and rhetoric in mathematics. The development of philosophical recognition of the social context of mathematics can be traced as part of the diminished domination of mathematical field by complete circle of philosophies (Reid et al., 2005; Smith et al., 2005). The epistemological perspective encompasses all knowledge that must have a certain warrant, validated by higher learning institutions that has a huge demand of mathematical philosophy. Thus, it enabled a parallel learning environment drawn with knowledge justification and learning assessment to permit the means of communicating mathematics to engineering educational students. The effect of educational developments is either direct or indirect, as they do not lead to immediate logical implications for mathematics teaching and learning without the curriculum assumptions (Davis et al., 2006; Dekkers et al., 2011).

Nevertheless, investigating the body of mathematical knowledge as a means of inquiry in mathematics and engineering education recalls the philosophies of mathematics that are central to mathematical multiplicity of theoretical perspectives rooted in radical constructivism (von Glasersfeld 1995), social constructivism (Piaget, 1971; Ernest 1991), and socio-cultural views (Lerman 1994) that has classroom concerns. Recent development around the interpretation of the social context and professional communities of mathematicians has a central role to play in the creation and justification of mathematical knowledge (Dym et al., 2005; Ernest, 2018). Thus, these communities are not merely organisations of individuals that are incidental to mathematics education but rather they play an essential role in epistemology through social organization and structure that is central to the mechanisms of mathematical knowledge generation and justification. These two ways are the repositories and sites of application and transmission of tacit and implicit knowledge in education, in which the vital roles played by social and cultural contexts, having impact on the centrality of the inferred and inherent mathematical knowledge in engineering mathematics, which has no basis for arguments as they are extensively well-known (de loss Rios et al., 2010; Wedding et al., 2018).

In addition, the interpretation of the philosophy of mathematics education raises the issues of teaching and learning of mathematics, with its underlying aims and rationales of the roles played by educators and students, as well as the underlying core values of its relevance in engineering education. To a greater extent, the curriculum theorist Joseph Schwab (1961) as cited by Ernest (2018), highlighted four common places of mathematics education teaching: curriculum basics, mathematics as a subject, faculty and students, and teaching background which include the link of teaching and learning as well as its objectives to humanity in broad-spectrum. Thus, a broader interpretation of the philosophy of mathematics education displays mathematical issues to the fore from a philosophical perspective, as one can apply philosophy to mathematics education that explores specific implications of philosophy of engineering education application (Tall, 2008; Wilkins et al., 2018). So, the philosophy of mathematics education should be emphasized to include the application of philosophical concepts and methods in the conceptual analyses of the outcomes of mathematics education research, and mathematics itself (Nye et al., 2018; Li et al., 2019a, 2019b). Hence, mathematics education is special field of research associated with public policy that concerns educational practices of teaching mathematics that has a central role to play in the philosophy of mathematics education.

Social philosophy and ethical dimensions of mathematics education with its associated benefits

The social philosophy and ethical dimensions of mathematics education with its associated benefits are linked to mathematical applications to real life situations that are traditionally been theorized as a form of transferring knowledge from one context to another (Avital et al., 1968; Turns et al., 2014; Marshall et al., 2017). The concept of knowledge transfer is founded in cognitive theories of learning that posits as developmental models of skill acquisition. This approach has provided mathematical application a platform of problem solving, which is theorized as higher-level skills involving transferable basic skills (Li, 2018a; 2018b). These basic skills are successfully un-situated and internalized to new and unfamiliar situations that brings challenges to the new concept of knowledge transfer. The transfer of knowledge usually come from ethnographers who have studied the context-dependent nature of cognition, which argues that cognition and skill acquisition cannot be abstracted from socio-cultural context. However, doing mathematics is deeply embodied in cultural practices and its applications are complex in social enactments of identity which has an implication on engineering educational curriculum and programmes (Borko et al., 2017; Blotnicky et al., 2018).

Thus, ethics comes into play in mathematics educational research in a number of ways including addressing a concern with values, social justice, and equity approaches, through research methodological ethics. Several studies have argued that despite its traditional value-free absolutist image and social identity, mathematics is value laden (Aiken, 1972; Bauersfeld, 1992; Berg, 1994; Chapman, 1995). While other studies have linked it to the concept of emancipatory philosophy, which argues that learning mathematics can be a revolutionary activity and should be based on emancipatory and empowerment of knowledge through fostering critical model applications from different subject domains (Sun et al., 2018; Ernest, 2018; Trenholm et al., 2019). Another prevailing strand of ethics-driven research in mathematics education concerns social justice and its deficiencies in engineering education of special groups such as gender, ethnic minorities, individuals with special needs and disabilities, language learners, and individual lower socio-economic status. These stated concerns spawned a vast literature and research that is labelled as social and political expertise which has an overt philosophical dimension, that predominantly focuses on the ethics of exclusion or disadvantage.

Ethical dimensions in mathematics and engineering education has called for morality, moral values and phenomena as well as criteria of morality for its object of research (Trenholm et al., 2019; Wedding et al., 2018). As a philosophical subject, ethics can be defined as a theory of morality which is covered by two aspects of notion such as the specific quality that recognizes the domain of its possible applications and part of the socio-cultural reality. Mathematics education seems bereft of ethical principles that might incite moral outrage in the areas of theoretical prepositions and solving skills platforms (Sun et al., 2018; Trenholm et al., 2019). Since ethical issues are often the source of action, one can draw connections between the processes of ethical filtration in mathematics education and the process of disengagement in mathematics school of learning. It has been recalled that mathematics textbooks have failed to offer students the controversial problems in applying mathematical skills as educators often do not have alternative resources to introduce current issues into the curriculum prospectus. More often, students perceive mathematics discipline as being disconnected from the world of language and politics, precisely, owing to mathematics textbooks offers limited sense of contextual relevance (Hayward et al., 2018; Nye et al., 2018).

The relationships between mathematics, mathematics education and issues such as social justice and equity have been addressed by the socio-political tradition in mathematics education. Others (Wilkins et al., 2018; Trenholm et al., 2019) have introduced explicit discussion of ethics, advocating for its centrality. However, this is an area that is still underdeveloped. Mathematics educators can make ethical choices that are not necessarily ambiguous and complex, but with illustrated examples from practices. The concept of ethical dimension is introduced as a heuristic to consider the awareness of different forms of relationship and arenas of action (Ernest, 1994; 2018; Li et al., 2019a; 2019b). A framework is proposed and discussed of four important dimensions: the relationship with others, the societal and cultural, the ecological and the relationship with self. Attending to the different ethical dimensions supports the development of a plural relational ethics. An ethics that takes account of these different dimensions supports an ethical praxis that is based on principles of flexibility and a dialogical relationship to the world and practice. Hence, navigating ethical complexity requires embracing diverse and changing commitments towards mathematical philosophy.

IV. THEORTICAL FRAMEWORK

This paper was guided by Jean Piaget's theory of the social constructivism, which was adapted and applied to interpret the ideals of the philosophical dimensions of mathematics in engineering education. The social constructivism explains that the dimensions of mathematics philosophy are primarily seen as a social construct and a product of culture, subjected to correction and change (Piaget, 1971). Unlike other sciences disciplines, mathematics is viewed as an empirical endeavour whose outcomes are constantly assessed and sometimes, may be discarded as required. Notwithstanding, an empiricist view on the assessment of some sort of comparison with 'reality', social constructivists emphasizes that the direction of mathematical research is dictated by the patterns of the social group performing it or by the needs of the society financing it (von Glasersfeld, 1995; Piaget, 1971; Ernest, 1998). Although, such external forces may cause changes that will affect the direction of some mathematical research, there are strong internal constraints such as mathematical traditions, methods, problems meanings and values into which mathematicians are enculturated. This implies that the philosophy of mathematics works to conserve a historically defined knowledge domain (von Glasersfeld, 1995; Gamoran et al., 2000; Nola et al., 2006). This domain therefore becomes imperative as it runs counter to the traditional beliefs of working mathematicians; having a critical thinking of the trend of how mathematics subject areas is considered to be pure or objective.

Also, social constructivists argued that mathematics is considerably grounded by much precariousness, as mathematical practice evolves the status of previous scientific disciplines is cast into doubt. This presumption can be corrected to a degree if it is required or desired by the current mathematical community to illustrate and dwell more on the philosophical dimensions involved in new mathematical theorems and models (Prawat et al., 1994; von Glasersfeld, 1998). Thus, this can be viewed from the perspective of the analytical development from re-examination of the calculus of Leibniz and Newton. These mathematical Nobel Lauren further argued that a finished mathematics is often accorded too much status, and folk mathematics is not enough, owing to an overemphasis on axiomatic proof alongside with peer review as practices (Piaget, 1971; Gamoran et al., 2000; Leask et al., 2001a). Thus, social constructivists have argued that each specialty forms its own epistemic gathering of knowledge-based domain, often with great difficulty in communicating, or motivating core investigation of unifying conjectures that are related to different areas of mathematics (Prawat et al., 1994; von Glasersfeld, 1995; Nola et al., 2006). Hence, the process of 'doing mathematics' is actually viewed as creating a meaning from social constructivists perspectives while social realists see a deficiency either of human capacity to abstraction, or of human's cognitive bias as well as mathematicians' collective intelligence as preventing the comprehension of a real universe of mathematical objects.

V. DISCUSSION

The dimensions of the philosophy of mathematics in engineering education can be presented as a research field as well as its value for curriculum, instruction, and teacher pedagogy. This paper seeks to for faculty and professionals to have a re-think on mathematics education, as an educational endeavour that examines past reform efforts, that have been partially seen as successful, including fundamental goal of achieving scientific literacy after several 'reform waves' has proven to be so elusive (Ernest, 2018; Li, 2018a; 2018b; Trenholm et al., 2019). Thus, the identity of such a philosophy is first defined in relation to the fields of philosophy, philosophy of science, and philosophy of education. Arguably, educational theory can support teacher's pedagogical knowledge content and history, philosophy and sociology of science should inform and influence pedagogy. This contribution to mathematics education seeks in general to the improvement of curriculum design and educators (Beumann et al., 2018; Li et al., 2019a; 2019b). While engineering is a science that focuses on forces of nature that are converted for mankind utility, as natural forces, mathematical analysis are compared with individuals' benefits. The consequences of mathematics and science in engineering has revealed that these subjects are major integral parts of engineering. Science focuses and teaches laws of the natural world and mathematics helps in establishing relationships, therefore, both subjects are of paramount importance in studying engineering at the higher educational institutions (Zhao et al., 2016; Wilkins et al., 2018).

From the elementary level, awareness of science, technology, engineering, and mathematics (STEM) fields are included in school curriculum. Apart from reading these courses, students are made aware of the fields and scope of STEM subjects. Students are made to study these subjects rigorously in order to prepare for mathematical-related courses as they pass on to higher learning from high school. Considering engineering as an option after secondary school level, prepares students to deal with a lot of mathematics and science, in which, most of the subjects are part of engineering courses (Harris et al., 2014; Marshall et al., 2017). Students are typically made to plan for careers related to science and mathematics, as they are crucial in preparing for advanced university programmes. As examination assessments remains, almost the same across globe, still requires the need to be proficient in science and mathematics subjects. Therefore, the prescribed syllabus and curriculum programmes in engineering entrance examinations are similar in focusing on clear assessments of understanding basic concepts and subjects. A major indicator of vast aptitude in studying engineering in higher institutions requires knowledge and competence in mathematical solving problems in engineering education (Bauersfeld, 1992; Li et al., 2019a; 2019b).

Globally, a lot of debates on engineering education has attracted meetings and symposiums where engineering is seen as a practical and handy jobs, with strong vision of engineering practices that collaborate and work with other professionals and teams to achieve things that work better in engineering institutions. Consequently, undergraduate students rightfully envisage that hands-on practical experiences with tangible engineering systems and working in collaboration teams are part of engineering experiences (Huang et al., 2011; Jacobs et al., 2017; Sun et al., 2018). Engineering subjects at most elite institutions require students to learn a large amount of mathematical theory involving learning of an advanced and complex practical component in mathematics. Thus, traditional, and conventional teaching of engineering mathematics encourages students to learn in isolation by repetition, echoing and memorizing steps taken by engineering educators to solve theoretical questions in applied mathematics (Austin et al., 1979; Kilgore et al., 2013; Borba et al., 2016). These mathematical models directly contradict the vision of activities in engineering education created by students and educators. Sometimes, mathematics is usually taught 'out of context', without any proper reference to the underlying practical engineering problems, as a result many engineering educational students lose motivation that leads to poor comprehension of fundamental mathematics in engineering. Hence, without a strong mathematics background, engineering students usually have a great level of difficulty in engineering courses, ultimately, resulting in a very low level of satisfaction with a complete engineering programme (Cobb, 1986; 1989; Quinn et al., 2015; Liljedah et al., 2016).

Besides, the structure of the philosophy and dimensions of mathematics education in engineering curriculum is important and must be discussed in educational boards of advisory. Traditionally, mathematics has been taught as a separate subject in engineering courses in universities worldwide (Stanic et al., 1992; Dossey et al., 2016; Marshall et al., 2017). This is as a result of mathematics as a course is common across all engineering disciplines and it was assumed that mathematics teaching and learning would be more efficient if undergraduates from all engineering subject areas were to be categorised jointly to study mathematics. Though, at first glance this might seem like a logical approach which has raised few issues in the past (Nye et al., 2018; Li et al., 2019a; 2019b). First and foremost, isolating mathematics from all engineering disciplines, makes it difficult for students to view mathematics in context. Engineering students should begin to see that mathematics as a course is used to solve practical engineering problems. Accordingly, if mathematics is taught as a separate subject, then the starting point of orientation is typically the mathematical equation and not necessarily the underlying engineering problem (Freeman et al., 2014; Blotnicky et al., 2018). Consequently, many students have failed to see the relevance of learning mathematics and another issue that needs to be considered is mathematics requirement of different engineering disciplines must be looked into.

Different engineering disciplines require knowledge of different mathematics topic and the level of knowledge in each mathematics subject areas are also distinct from each engineering discipline. Thus, if all engineering students were required to assume usual mathematics subjects, which demonstrates that engineering students will be learning mathematics that they do not need in their entire engineering career (Ma et al., 2004; Quinn et al., 2015; Li et al., 2019). Another shortcoming of teaching mathematics subjects to all engineering students is that class sizes are generally very large and clumsy, together with mathematics usually taught in a didactic way. This reflects an undesirable situation as it is not conducive for a good learning environment for engineering educational students. Also, the reason behind isolating mathematics teaching from engineering science might seem logical from an administrative perspective but lacks pedagogical benefits as far as undergraduates are concerned (Reid et al., 2005; Tall, 2008; Trenholm et al., 2019). On the basis of this argument, it would seem logical to simply incorporate mathematics teaching into teaching of engineering science subjects of the different disciplines of engineering.

Conversely, there are some major concerns engineering educators need to take cognizance of, these issues created a serious gap in engineering education. First, numerous mathematics themes are needed across many engineering disciplines and, thus, if different engineering educators teach the same topics in dissimilar engineering subjects, it could create a position where teaching becomes monotonous in engineering classes (Schoenfeld, 1992; Rotman, 1988; Marshall et al., 2017; Wilkins et al., 2018). This would merely be a waste of time and resources. Second, there is a real danger for students to think that a particular mathematics topic, is only relevant to a particular engineering subject that is taught (Chapman, 1997; Dekkers et al., 2011; Wedding et al., 2018). Notwithstanding, with these concerns, there have been efforts to increase undergraduates' engagement with mathematics by incorporating it into engineering science subjects. This strategy have been developed and offered in engineering higher learning of institutions, as it aims to expose undergraduates early to mathematics in engineering (Dossey et al., 2016; Nye et al., 2018; Trenholm et al., 2019). Thus, the idea of integrating mathematics into engineering teaching and subjects is rather innovative but has a strong support for educators in leading engineering schools to the 21st century skills. Hence, this integration of mathematics in engineering subjects will meet desired outcomes in motivating students to gain a deeper understanding of mathematics in engineering education.

Implications for Practice

Engineering as an important core subject in mathematical discipline, has stimulated new areas of development in mathematical research. Mathematics has a vital role to play in engineering education and has taken cognizance of the association between engineering and mathematics education. In the last decades, new demands of engineering and inadequate mathematics become important to acknowledge as 21st century skills require huge demands from engineering graduates' students. Recently, the dawn of technology and sophisticated computer developments have led to disparity in mathematics teaching in engineering institutions, helping students to equip with modern use of techniques and methods. Insufficient skills in basic mathematics can produce difficulties for those majoring in engineering, as huge proportion of engineering students seems to find accurate solution in testing and examining engineering mathematical-related questions that feature common steps and methods. Yet, they lack deep theoretical understanding of the essential theorems and occasionally have misconstructions. To address these misrepresentations in mathematics, engineering students are embedded to develop themselves in areas of problem solving and creative thinking in engineering fields. This speaks to the importance of building engineering students with serious mathematics education that will build students with the modern skills and abilities that is aligned with knowledge and competencies. Though, the significance of mathematics education in engineering have been cited in many studies (Jacobs et al., 2017; Ernest, 2018) as there are no consensus on the volume and content of mathematics that is required for engineering students. But a consensus on the need for a basic mathematics in all engineering disciplines can be emphasized and be streamlined in engineering programmes and curriculum respectively. This is the most important and effective methods that will aid in building engineering mathematics in the programmes that be more beneficial to students. Also, this will give students the chance to see the main developments in the concepts and understanding of core mathematical related subjects in engineering programmes. Thus, mathematics should be included in different engineering curriculum and should be revised in line with the contemporary prospectus that will meet the basic requirements of the 21st century scientific and technology jobs.

VI. Conclusion and Limitations

Engineering education is expected to provide hard and soft skills to engineering students to form competent human resources so as to compete with their counterparts internationally. Therefore, it takes a variety of efforts to realize these expectations as most important aspect in a country's development is education. Education, which is needed nowadays, is an education that is able to provide undergraduates with the basic learning experiences to enhance students' abilities in solving problems, thinking deeply, managing technological projects, and using various technology and information tools. Conversely, there are quite a few subjects that are important in learning as their significance is seen in in real life situations and in combination with other disciplines. Thus, one of the most important subjects to learn in engineering schools and across other disciplines is mathematics, as it is a core subject that is applied to everyday life situation.

Also, mathematics subject can be integrated with various disciplines and the 21st century era demands competent human resources in Science, Technology, Engineering and Mathematics (STEM). With regard to economic growth of the 21st century, labour force must have the skills of science and mathematics, creativity, expertise in information and communication technology, and the ability to solve complex problems. In order to compete in the global economic system of the 21st century, a nation must establish an education where students can gain an understanding of STEM including computer technology and produce the product using the required skills in the field. This becomes an imperative as engineering educators and students as well as professional engineers must have to think mathematically and to use mathematics to describe and analyse different aspects of the real world where they seek to engineer. Thus, mathematical modelling therefore plays a key role in the formation of engineers, and there has been much research into how engineers should be taught the essential mathematics. This implied that engineers should be involved in mathematics teaching to engineering students, as to oversee how mathematicians are teaching engineering students mathematics. Hence, closing the gap in the philosophical dimensions of mathematics in Engineering education, is of critical importance in engineering institutions as failure to address these concerns in engineering mathematics education will result to wider disparities in the production of engineering graduates. Therefore, key recommendations are advocated as follows:

1. Faculty capacity building and self-developments should be encouraged in line with modern mathematics and engineering contemporary programmes so they can be able to deliver their teaching methods that will be beneficial to engineering students.
2. Restructuring mathematics in engineering education in line with contemporary curriculum will aid in teaching and learning methods that will equip students with innovative and better mathematical learning tools that will prepare them for the 21st century era.
3. Engineering and mathematics learning institutions and universities should be funded in areas of providing contemporary mathematical textbooks, research tools and e-classrooms that will expose undergraduates' students to better learning facilities and platforms.

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