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Isotropic Perfect Fluids in Modified Gravity

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Abstract: We generate the Einstein–Gauss–Bonnet field equations in higher dimensions for a spherically symmetric static spacetime. The matter distribution is a neutral fluid with isotropic pressure. The condition of isotropic pressure, an Abel differential equation of the second kind, is transformed to a first order nonlinear canonical differential equation. This provides a mechanism to generate exact solutions systematically in higher dimensions. Our solution generating algorithm is a different approach from those considered earlier. We show that a specific choice of one potential leads to a new solution for the second potential for all spacetime dimensions. Several other families of exact solutions to the condition of pressure isotropy are found for all spacetime dimensions. Earlier results are regained from our treatments. The difference with general relativity is highlighted in our study.

Keywords: Einstein–Gauss–Bonnet gravity; relativisitic fluids; static metrics

1. Introduction

Exact solutions in general relativity have been intensively researched over the years and applied to astrophysics and cosmology. A useful guide to physically relevant solutions is contained in Stephani et al. [1]. Spacetimes with spherical symmetry have received particular interest as they may be applied to static [2,3] and nonstatic spheres [4]. There has been much interest in extending such studies to modified theories of gravity such as $f(R)$, $f(R,T)$ and $f(T,B)$ gravity, amongst others. A case that received recent attention is $f(R,T,Q)$ gravity to explain the extra gravity that needs to be assigned to dark matter [4–9]. The Lanczos–Lovelock (or Lovelock) models are a natural generalisation of general relativity in higher dimensions containing the generalised Lovelock tensor with curvature corrections [10]. They have special significance in studying and testing emergent phenomena in gravity. The first extension in Lovelock gravity, extending general relativity, is the second order theory, namely the Einstein–Gauss–Bonnet (EGB) Lagrangian. EGB gravity lends itself to the study of important physical phenomena such as thermodynamics of horizons and gravitational collapse [11–14]. The EGB theory with static spherical symmetry is the objective of this study. We show that exact solutions in this modified theory of gravity can be obtained. Our models contain earlier results and have interesting physical properties. The approach employed in this paper can be extended to the other modified theories of gravity mentioned above.

The gravitational dynamics of localized distributions are affected by higher order curvature corrections which generalize general relativity. The Lagrangian of general relativity has to be extended to include terms which are products of the Ricci scalar, the Ricci tensor and the Riemann tensor. Einstein–Gauss–Bonnet gravity (EGB) gravity is the most promising higher order curvature theory because of its consistency with observations, inherent geometrical features and compatibility with the required physics. EGB gravity reduces to general relativity in the relevant limit, and it is part of the more general Lovelock class of gravity theories. The important respective physical processes of dissipation and gravitational collapse [15,16] influence collapsing spheres in EGB gravity. Various physical aspects of compact bodies have been investigated by several authors [17–23]. Strange star

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models have been recently generated by Jasim et al. [24] and Maurya et al. [25]. Quark matter distributions in EGB gravity have also been found [26,27]. It is important for certain astrophysical processes that a polytropic equation of state be satisfied. Polytropic stars have been found by Maurya et al. [28] and Kaisavelu et al. [29]. The models mentioned above demonstrate features which are different from general relativity.

For a study of the physical features in stellar models it is necessary to find exact solutions to the EGB field equations. Particular classes of exact solutions in static metrics have been found mainly in five and six spacetime dimensions [30–35] for neutral matter distributions with isotropic pressure. Other interesting models have been studied by [36–39]. Consequently in these investigations mentioned above the condition of pressure isotropy is satisfied. We will require that pressure isotropy be a necessary condition in our treatment, as this physical requirement is normally required in self-gravitating bodies; the connection with general relativity is made easier as most of the familiar Einstein stellar models are isotropic. Note that anisotropic pressures are possible, and physically viable models have also been found in EGB gravity. Anisotropic models are easier to generate as the EGB system of field equations may be then considered as an algebraic system of equations with more unknowns than equations; there is no differential equation corresponding to the pressure isotropy condition to solve.

The Pascal principle is equivalent to the local isotropy of pressure condition, and it has been an assumption used in many studies of self-gravitating objects. The condition of pressure isotropy is supported by observational evidence in different physical scenarios. Therefore the Pascal principle is an assumption that should be investigated in modified gravity theories since they provide a good explanation of the observed expansion of the universe. However there are two issues that need to be highlighted about the condition of pressure isotropy arising from fluctuations in physical systems. Firstly pressure anisotropy is produced by physical processes in highly compact bodies as shown by Herrera et al. [40]. Small amounts of anisotropy may lead to fractures and cracking of the fluid body [41]. This indicates that the stability of the self-gravitating sphere has to be analysed. Secondly the presence of pressure anisotropy acquired during the dynamic regime of collapse may not disappear in the final equilibrium state. An initial system with isotropic pressures will lead to a final configuration with anisotropy as established by Herrera [42]. We point out that there is no known physical process that enables the anisotropy to be eliminated leading to a final isotropic state. The physical features mentioned above have been established in general relativity. It is important that the possibility of instability be also considered in EGB gravity and to then pursue the physical implications. In EGB gravity the presence of the Lovelock tensor contributes higher order curvature corrections to the field equations. These additional curvature terms are absent in general relativity. The new curvature terms in EGB gravity will definitely affect the dynamics of the gravitating fluid and will consequently influence the stability of the body. This is an important issue and will be studied in future research.

In an ideal situation it would be desirable to have an algorithm that generates EGB solutions in a systematic fashion. In general relativity particular algorithms are known to exist of varying utility [43–51]. The situation in EGB gravity is more difficult as the pressure isotropy condition is an Abel differential equation of the second kind. Maharaj et al. [52] presented a solution generating algorithm which yields a new solution from an existing known solution. In this paper we follow a different route. The condition of pressure isotropy is derived in N dimensions in EGB gravity for static spherical metrics. This pressure isotropy condition is an Abel differential equation of the second kind [53] in the potentials y and Z. We show that this Abel differential equation can be transformed to the canonical form $w\dot{w} = F_1 w + F_0$ which is a simpler nonlinear differential equation. Then any choice of the potential y leads to a solution yielding Z. We demonstrate the efficacy of this new solution-generating algorithm by making a specific choice of y. A new solution in EGB gravity is found for all spacetime dimensions N. Other classes of new exact solutions are also found for the canonical differential equation. Existing solutions are regained from our treatments.

The analysis in this paper shows that general relativity is special. The field equations yield a linear equation in Z for isotropic pressures. In EGB gravity the condition of pressure isotropy is nonlinear in the potential Z which allows for a new solution generating algorithm to be proposed. This algorithm does not arise in general relativity.

2. EGB Gravity

An action will generate field equations in any gravity theory. The Einstein–Hilbert action generates the well known Einstein field equations in any dimension; a modification of this very action results in the Gauss–Bonnet action that is required to produce the EGB field equations. An interesting feature of this action is that the field equations appear as second order differential equations which are quasilinear in the highest derivative. In addition, the action is valid in arbitrary spacetime dimensions. The Lovelock tensor H_{ab} is described by

$$H_{ab} = g_{ab}L_{GB} - 4RR_{ab} + 8R_{ac}R^{c}{}_{b} + 8R^{cd}R_{acbd} - 4R_{a}{}^{cde}R_{bcde}, \tag{1}$$

and the Gauss–Bonnet term is written as

$$L_{GB} = R^2 + R_{abcd}R^{abcd} - 4R_{cd}R^{cd}. \tag{2}$$

The EGB field equations with matter are derived in the form

$$G_{ab} - \frac{\alpha}{2}H_{ab} = \kappa_N T_{ab}. \tag{3}$$

In the above, G_{ab} is the Einstein tensor, α is the Gauss–Bonnet coupling constant, κ_N is the gravitational coupling constant defined by

$$\kappa_N = \frac{2(N-2)\pi^{\frac{N-1}{2}}G}{c^4(N-3)\Gamma\left(\frac{N-1}{2}\right)}, \tag{4}$$

and T_{ab} is the energy momentum tensor.

3. Higher-Dimensional Spherical Model

The line element for a spherically symmetric static spacetime in N dimensions is given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\Omega^2_{N-2}, \tag{5}$$

where $\nu(r)$ and $\lambda(r)$ are arbitrary functions of r that represent gravitational potentials, and the $(N-2)$-sphere is denoted by

$$d\Omega^2_{N-2} \quad = \quad \sum_{i=1}^{N-2}\left(\left[\prod_{j=1}^{i-1}\sin^2(\theta_j)\right](d\theta_i)^2\right). \tag{6}$$

The energy momentum tensor for uncharged matter is defined by the symmetric tensor

$$T_{ab} \quad = \quad (\rho + p)u_a u_b + p g_{ab}, \tag{7}$$

where ρ is the energy density and p is the isotropic pressure. All these quantities are measured relative to a comoving fluid velocity $\boldsymbol{u}$ which is unit and timelike $(u^a u_a = -1)$. Equating the curvature components to the matter components using the relationship (3), we can obtain the EGB field equations in N dimensions. These are expressed by

$$\begin{aligned}
\kappa_N\rho &= \frac{(N-2)}{r^4e^{4\lambda}}\left[r^3e^{2\lambda}\lambda' + \frac{(N-3)r^2e^{4\lambda}}{2} - \frac{(N-3)r^2e^{2\lambda}}{2}\right.\\
&\left.+\alpha(N-3)(N-4)\left(e^{2\lambda}-1\right)\left(2r\lambda' + \frac{(N-5)\left(e^{2\lambda}-1\right)}{2}\right)\right], \qquad (8a)\\
\kappa_N p &= \frac{(N-2)}{r^4e^{4\lambda}}\left[r^3e^{2\lambda}\nu' + \frac{(N-3)r^2e^{2\lambda}}{2} - \frac{(N-3)r^2e^{4\lambda}}{2}\right.\\
&\left.+\alpha(N-3)(N-4)\left(e^{2\lambda}-1\right)\left(2r\nu' - \frac{(N-5)\left(e^{2\lambda}-1\right)}{2}\right)\right], \qquad (8b)\\
\kappa_N p &= \frac{1}{r^2e^{2\lambda}}\left[\frac{(N-3)(N-4)}{2} + r^2\nu'' + r^2\nu'^2 - r^2\nu'\lambda' + (N-3)r(\nu'-\lambda')\right.\\
&\left.+2\alpha(N-3)(N-4)\left(\nu''+\nu'^2-\nu'\lambda'\right)\right] + \frac{(N-3)(N-4)}{r^2e^{4\lambda}}\left[6\alpha\nu'\lambda'\right.\\
&\left.-2\alpha\left(\nu''+\nu'^2\right) - \frac{e^{4\lambda}}{2}\right] + 2\alpha(N-3)(N-4)\left[\frac{(N-5)}{r^3e^{4\lambda}}\left(e^{2\lambda}-1\right)(\nu'-\lambda')\right]\\
&-\frac{\alpha(N-3)(N-4)(N-5)(N-6)\left(e^{2\lambda}-1\right)^2}{2r^4e^{4\lambda}}, \qquad (8c)
\end{aligned}$$

in terms of Schwarzschild coordinates. An outline of the derivation of the EGB field equations is given in the Appendix A.

In order to simplify system (8), we apply the transformation

$$e^{2\nu(r)} = y^2(x), \quad e^{-2\lambda(r)} = Z(x), \quad x = r^2, \qquad (9)$$

first introduced by Durgapal and Bannerji [54] in general relativity. The EGB field equations with isotropic pressure can then be written as

$$\begin{aligned}
\kappa_N\rho &= (N-2)\left[\frac{(N-3)(1-Z)-2x\dot{Z}}{2x}\right.\\
&\left.+\frac{\alpha(N-3)(N-4)(1-Z)}{2x^2}\left(-4x\dot{Z}+(N-5)(1-Z)\right)\right], \qquad (10a)\\
\kappa_N p &= (N-2)\left[\frac{2Z\dot{y}}{y} + \frac{(N-3)(Z-1)}{2x} + \alpha(N-3)(N-4)(1-Z)\right.\\
&\left.\times\left(\frac{4Z\dot{y}}{xy} - \frac{(N-5)(1-Z)}{2x^2}\right)\right], \qquad (10b)\\
\kappa_N p &= \frac{2}{y}\left[2xZ\ddot{y} + x\dot{Z}\dot{y} + (N-2)\dot{y}Z\right] + (N-3)\left[\dot{Z} + \frac{(N-4)(Z-1)}{2x}\right]\\
&+\alpha(N-3)(N-4)\left[\frac{4(N-4)Z(1-Z)\dot{y}}{xy} + \frac{8Z(1-Z)\ddot{y}}{y}\right.\\
&\left.+\frac{4\dot{Z}\dot{y}(1-3Z)}{y} + \frac{2(N-5)\dot{Z}(1-Z)}{x} - \frac{(N-5)(N-6)(1-Z)^2}{2x^2}\right]. \qquad (10c)
\end{aligned}$$

Equating (10b) and (10c) yields the isotropic pressure condition

$$
\begin{aligned}
&4x^2 Z\left[x+2\alpha(N-3)(N-4)(1-Z)\right]\ddot{y} \\
&+2x\left[x^2\dot{Z}-2\alpha(N-3)(N-4)\Big(2Z(1-Z)-x(1-3Z)\dot{Z}\Big)\right]\dot{y} \\
&+(N-3)(x\dot{Z}-Z+1)\Big(x+2\alpha(N-4)(N-5)(1-Z)\Big)y=0,
\end{aligned}
\tag{11}
$$

which determines the gravitational behaviour. When $N = 5$ the EGB field equations were first obtained by Maharaj et al. [30]. Note the appearance of the term $2\alpha(N-4)(N-5)(1-Z)$ in the coefficient of y. This term does not arise in the condition of pressure isotropy when $N = 5$. Hence the dimension N has a dramatic effect on the gravitational behaviour of the model for $N \geq 6$. Note that the condition of pressure isotropy for $N = 5$ was also presented by Hansraj et al. [31], and by Hansraj and Mkhize [33] when $N = 6$. Our result (11) holds for all dimensions N. Our intention is to find exact solutions for all N.

An equivalent form of (11) is given by

$$
\begin{aligned}
&\Big[2x^3\dot{y}+4\alpha(N-3)(N-4)x^2\dot{y}-12\alpha(N-3)(N-4)x^2\dot{y}Z+(N-3)x^2y \\
&+2\alpha(N-3)(N-4)(N-5)xy-2\alpha(N-3)(N-4)(N-5)xyZ\Big]\dot{Z} \\
&+2\alpha(N-3)(N-4)\left[4x\dot{y}-4x^2\ddot{y}+(N-5)y\right]Z^2 \\
&+\Big[4x^3\ddot{y}+8\alpha(N-3)(N-4)x^2\ddot{y}-8\alpha(N-3)(N-4)x\dot{y}-(N-3)xy \\
&-4\alpha(N-3)(N-4)(N-5)y\Big]Z+(N-3)xy \\
&+2\alpha(N-3)(N-4)(N-5)y=0.
\end{aligned}
\tag{12}
$$

The form (12) looks more complicated but it does have an advantage over (11). If the function y is specified then we can interpret (12) as a nonlinear first order differential equation in Z. This form allows for a general analysis as we demonstrate below.

4. The Einstein Case

In this case, set $\alpha = 0$ in (12), which becomes

$$
\left[2x^3\dot{y}+(N-3)x^2y\right]\dot{Z}+\left[4x^3\ddot{y}-(N-3)xy\right]Z+(N-3)xy=0. \tag{13}
$$

This differential equation is linear and first order in Z.

Several solutions to (13) have been found in four dimensions. In particular, when $y = \sqrt{x}$ then (13) yields the solution

$$
Z=\frac{N-3}{N-2}+C_1x, \tag{14}
$$

where C_1 is an integration constant. For a comprehensive list of exact solutions to (13) with $N = 4$, see Delgaty and Lake [55]. In higher dimensions $N \geq 5$ some exact models were found by Krori et al. [56] and Patel et al. [57].

We summarize the main result as

Theorem 1. *When $\alpha = 0$ the condition of pressure isotropy for the Einstein case is a first order linear differential equation in the potential Z in N dimensions. Any choice of the potential $y = y_0$ then generates the potential $Z = Z_0$.*

Corollary 1. *When $N = 4$ all exact solutions to the Einstein field equations with isotropic pressure are contained in* (13).

5. The EGB Case

When $\alpha \neq 0$, then (12) is no longer a linear differential equation in Z. It is a first order nonlinear ordinary differential equation in Z, and it is further identified as an Abel differential equation of the second kind. It can be simplified by applying a transformation as suggested in Polyanin and Zaitsev [53]. We introduce the new variable

$$w = \left(Z - \frac{(N-3)[2\alpha(N-4)(N-5)+x]y + 2x(x+2\alpha(N-3)(N-4))\dot{y}}{2\alpha(N-3)(N-4)[6x\dot{y}+(N-5)y]} \right) W, \qquad (15)$$

where

$$W = \exp\left(-\int \frac{(4x\dot{y} - 4x^2\ddot{y} + (N-5)y)}{x[6x\dot{y} + (N-5)y]}\, dx \right), \qquad (16)$$

and $N \geq 5$. Note that $\alpha \neq 0$ and $6x\dot{y} + (N-5)y \neq 0$. Equation (12) then reduces to

$$w\dot{w} = F_1 w + F_0, \qquad (17)$$

where $w = w(x)$ and we have introduced the new functions F_1 and F_0 which depend on the potential y. They have the forms given by

$$F_1 = \frac{x[y - 2(x+2\alpha(N-3)(N-4))\dot{y}][(N-3)\dot{y} - 2x\ddot{y}]W}{\alpha(N-3)(N-4)\left[6x\dot{y}+(N-5)y\right]^2}, \qquad (18)$$

and

$$\begin{aligned} F_0 = \; & x^2(-2y + 4(x+2\alpha(N-3)(N-4))\dot{y})[(N-3)y(\dot{y}+2(x+ \\ & 2\alpha(N-4)(N-5))\ddot{y}) + 2\dot{y}[(x-4\alpha(N-3)(N-4))\dot{y} \\ & +2x(x+2\alpha(N-3)(N-4))\ddot{y}]] \\ & \times \frac{W^2}{2\alpha^2(N-3)^2(N-4)^2[6x\dot{y}+(N-5)y]^3}. \end{aligned} \qquad (19)$$

Equation (17) is in canonical form but remains an Abelian equation. We need to show that exact solutions to (17) exist.

Theorem 2. *When $\alpha \neq 0$ and $6x\dot{y} + (N-5)y \neq 0$ the condition of pressure isotropy for the EGB case is a nonlinear Abelian differential equation of the second kind in Z in N dimensions. This Abelian differential equation can be transformed to the canonical form $w\dot{w} = F_1 w + F_0$.*

Corollary 2. *If a functional form for the potential $y = y_0$ is specified then $w\dot{w} = F_1 w + F_0$ can be integrated in principle to obtain the potential $Z = Z_0$.*

A Specific Potential

We demonstrate that a solution to the canonical Equation (17) exists by letting

$$y = \sqrt{x}. \qquad (20)$$

This choice was also made by Hansraj and Mkhize [33] for the particular spacetime dimension $N = 6$. The integral in (16) can be evaluated and we obtain

$$W = \frac{1}{x}. \qquad (21)$$

Then (17) becomes

$$w\dot{w} = -\frac{1}{(N-2)x^2}w - \frac{2}{(N-2)^2x^3}. \tag{22}$$

Interestingly the use of (20) reduces the functions F_1 and F_0 to simpler forms involving the variable x; however (17), and consequently (22), remain nonlinear ordinary differential equations. This equation can be simplified by making a new substitution

$$\mathcal{W}(x) = \frac{1}{2}(N-2)xw(x). \tag{23}$$

This transformation leads to

$$\frac{2\mathcal{W}\dot{\mathcal{W}}}{2\mathcal{W}^2 - \mathcal{W} - 1} = \frac{1}{x}, \tag{24}$$

in terms of the new variable $\mathcal{W}(x)$. The remarkable feature of (24) is that it is a separable equation. Consequently (24) can be integrated to give

$$(\mathcal{W}-1)^4(2\mathcal{W}+1)^2 = \frac{C_1x^6}{2}, \tag{25}$$

where $C_1 > 0$ represents an integration constant. We can then write the result (25), using the transformation (23), in terms of the variable $w(x)$ in the form

$$\left(\frac{(N-2)xw(x)}{2} - 1\right)^4\Big((N-2)xw(x)+1\Big)^2 - \frac{C_1x^6}{2} = 0. \tag{26}$$

Equation (26) is a sixth order polynomial in $w(x)$. It is difficult to find roots of such polynomials but in our case we can factorize (26) to obtain

$$\begin{aligned}&\left[\left(\frac{(N-2)xw(x)}{2} - 1\right)^2\Big((N-2)xw(x)+1\Big) - \sqrt{\frac{C_1}{2}}x^3\right]\\ &\times\left[\left(\frac{(N-2)xw(x)}{2} - 1\right)^2\Big((N-2)xw(x)+1\Big) + \sqrt{\frac{C_1}{2}}x^3\right] = 0.\end{aligned} \tag{27}$$

Equation (27) has six roots: four complex roots and two real roots. The real roots are given by

$$\begin{aligned}w(x) = \frac{1}{(N-2)x}&\left[\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{-\frac{1}{3}}\right.\\ &\left.+\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{\frac{1}{3}} + 1\right].\end{aligned} \tag{28}$$

Thus we have obtained the new general solution of (17) with $y = \sqrt{x}$ for *all* spacetime dimensions N.

Observe that with $y = \sqrt{x}$ we have

$$W = \frac{1}{x}$$

from (21). Then we obtain from (15) the potential Z in an explicit form

$$\begin{aligned} Z &= \frac{1}{N-2}\left[\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{-\frac{1}{3}} \right. \\ &\quad \left. + \left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{\frac{1}{3}} + 1\right] \\ &\quad + \frac{(N-2)x + 2\alpha(N-3)(N-4)^2}{2\alpha(N-2)(N-3)(N-4)}. \end{aligned} \tag{29}$$

Hence we have demonstrated that the canonical Abel Equation (17) has the solution (20) and (29) for the EGB field equations for all dimensions $N \geq 5$. The result (29) represents a new class of solutions for the gravitational potential Z in N dimensions, in closed form. Our result shows that the gravitational potential Z can be written explicitly in terms of elementary functions. The dimension N has a direct effect on the potential Z.

We now consider some special cases arising from our new family of exact solutions. When $N = 5$ then (29) becomes

$$\begin{aligned} Z &= \frac{x}{4\alpha} + \frac{2}{3} + \frac{\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{-\frac{1}{3}}}{3} \\ &\quad + \frac{\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{\frac{1}{3}}}{3}. \end{aligned} \tag{30}$$

This is also a new solution and does not appear to be contained in earlier works. When $N = 6$, then (29) has the form

$$\begin{aligned} Z &= \frac{x}{12\alpha} + \frac{3}{4} + \frac{\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{-\frac{1}{3}}}{4} \\ &\quad + \frac{\left(-1 \pm \sqrt{2C_1}x^3 + \sqrt{\mp 2\sqrt{2C_1}x^3 + 2C_1x^6}\right)^{\frac{1}{3}}}{4}. \end{aligned} \tag{31}$$

Note that the special case (31) with $N = 6$ regains the exact solution of Hansraj and Mkhize [33]. Note that we have corrected some typographical errors in [33].

6. Exceptional Potentials

The transformation (15) holds when

$$\begin{aligned} \alpha &\neq 0, \\ 6x\dot{y} + (N-5)y &\neq 0. \end{aligned}$$

These two cases constitute exceptional cases and need to be considered separately.

Firstly, when $\alpha = 0$ we can integrate the condition of pressure isotropy (12) and obtain the potential Z for any chosen form of y. When $y = \sqrt{x}$ we obtain the result (14).

Secondly, we need to consider

$$6x\dot{y} + (N-5)y = 0. \tag{32}$$

We can integrate (32) to obtain

$$y = Cx^{\frac{5-N}{6}}, \tag{33}$$

where C is a constant. For this form of y the condition of pressure isotropy (12) becomes

$$-6x\left[\alpha(N-3)(N-4)(N-5)+\frac{(N-2)x}{2}\right]\dot{Z}+\alpha(N-2)(N-3)(N-4)(N-5)Z^2$$
$$-(N-11)\left[\frac{(N-2)x}{2}+\alpha(N-3)(N-4)(N-5)\right]Z-9(N-3)\left[\frac{x}{2}+\alpha(N-4)(N-5)\right]=0. \tag{34}$$

When $N=5$, expression (34) is a linear differential equation in Z. We obtain the solution

$$Z = \tilde{C}x+1, \tag{35}$$

which is given in terms of elementary functions for $N=5$ and $\tilde{C}$ is an integration constant.

In this case we obtain the equation of state

$$\rho = 4\alpha\pi^2 p^2 - 2p, \tag{36}$$

with constant pressure. When $\alpha = 0$ then $p = -\frac{1}{2}\rho$ which is contained in the higher dimensional Einstein models found by Patel et al. [57]. In four dimensions we have that $p = -\frac{1}{3}\rho$ which is the Einstein static universe. The equation of state (36) therefore corresponds to the generalised Einstein static model in EGB theory when $N=5$. A similar result to (36) was also found in [32] using a different approach.

Equation (34) takes on a particularly simple form for a special value of the spacetime dimension N. When $N=11$, the linear term in Z vanishes. In this case (34) reduces to

$$-6x\left[336\alpha+\frac{9}{2}x\right]\dot{Z}+3024\alpha Z^2-72\left[\frac{x}{2}+42\alpha\right] = 0, \tag{37}$$

which remains a Riccati equation in Z. The solution is given in closed form as

$$\begin{aligned} Z = & \left[-12\mathcal{C}(x+12\alpha)\left(x+\frac{224\alpha}{3}\right)\ln\left(\frac{224\alpha+3x}{224\alpha}\right)-25088\alpha^2\right. \\ & \left.+(1344\mathcal{C}-560)\alpha x+21\left(\mathcal{C}-\frac{1}{7}\right)x^2\right] \\ & \times\left[-1344\alpha\mathcal{C}\left(x+\frac{224\alpha}{3}\right)\ln\left(\frac{224\alpha+3x}{224\alpha}\right)-25088\alpha^2+(1344\mathcal{C}-336)\alpha x+9\mathcal{C}x^2\right]^{-1}, \end{aligned} \tag{38}$$

where $\mathcal{C}$ is a constant of integration.

The differential Equation (34) is a Riccati equation for all values of $N>5$. It can be integrated to give

$$
\begin{aligned}
Z \quad = \quad & \bigg[-18(N-3)\mathcal{C}\left(\left(\frac{2N-4}{3}\right)x^{\frac{5}{2}}+\alpha(N-3)(N-4)(N-5)x^{\frac{3}{2}}\right) \\
& \times\alpha(N-4)(13+N)(N-5)\text{Hypergeometric}\left(\left[2,\frac{N+7}{6}\right],\left[\frac{N+13}{6}\right],\right. \\
& \left.-\frac{(N-2)x}{2\alpha(N-3)(N-4)(N-5)}\right)+2(N-2)\left(6\mathcal{C}\left(\frac{(N-2)x^{\frac{7}{2}}}{2}\right.\right. \\
& \left.+\alpha(N-3)(N-4)(N-5)x^{\frac{5}{2}}\right)(N+7)\text{Hypergeometric}\left(\left[3,\frac{N+13}{6}\right],\left[\frac{N+19}{6}\right],\right. \\
& \left.-\frac{(N-2)x}{2\alpha(N-3)(N-4)(N-5)}\right)+\alpha(N-3)(N-4)(N-5)^2(N+13) \\
& \left.\times\left(\alpha(N-3)(N-4)x^{\frac{2-N}{6}}+\frac{x^{\frac{8-N}{6}}}{2}\right)\right)\bigg] \\
& \times\left(2\alpha^2(N-2)(N-3)^2(N-4)^2(N-5)^2(N+13)\right)^{-1} \\
& \times\left(\text{Hypergeometric}\left(\left[2,\frac{N+7}{6}\right],\left[\frac{N+13}{6}\right],-\frac{(N-2)x}{2\alpha(N-3)(N-4)(N-5)}\right)\right)^{-1} \\
& \times\left(\mathcal{C}x^{\frac{3}{2}}+x^{\frac{2-N}{6}}\right)^{-1}. \qquad (39)
\end{aligned}
$$

The solution when $N > 5$ is given in terms of both elementary functions and hypergeometric functions depending on the value of N. There is no simple equation of state relating the pressure p to the energy density ρ. This is different from the case $N = 5$ considered above. Again we see that the dimension N affects the dynamics of the model.

We believe that (33), together with (38) and (39), are new solutions to the EGB field equations. Therefore the cases $\alpha = 0$ and $6x\dot{y} + (N-5)y = 0$ can also be solved to generate new solutions.

7. Special Canonical Equation

We have transformed the condition of pressure isotropy (12) to the canonical Equation (17). We have demonstrated that exact solutions to (17) exist. Note that the canonical Equation (17) can be solved by placing constraints on F_0 and F_1. Observe that by placing a condition on F_1 a special case in Theorem 2 arises where it is possible to integrate (17) without specifying the potential y. To see this we set

$$F_1 = 0. \qquad (40)$$

This is a nonlinear differential equation but it has the interesting structure

$$[y - 2(x + 2\alpha(N-3)(N-4))\dot{y}][(N-3)\dot{y} - 2x\ddot{y}] = 0. \qquad (41)$$

In the above equation, we observe that it is a product of a first order and second order linear ordinary differential equation. As a result, we can obtain two solutions for the variable $y(x)$.

It remains to find Z if the condition (40) holds. Equation (17) becomes

$$
\begin{aligned}
w\dot{w} \quad = \quad & x^2(-2y+4(x+2\alpha(N-3)(N-4))\dot{y})[(N-3)y(\dot{y}+2(x+ \\
& 2\alpha(N-4)(N-5))\ddot{y})+2\dot{y}[(x-4\alpha(N-3)(N-4))\dot{y} \\
& +2x(x+2\alpha(N-3)(N-4))\ddot{y}]] \\
& \times\frac{W^2}{2\alpha^2(N-3)^2(N-4)^2[6x\dot{y}+(N-5)y]^3}, \qquad (42)
\end{aligned}
$$

which is a separable equation. Integrating we obtain the result

$$\begin{aligned} w &= \pm\Bigg[2\int\Bigg(x^2\big(-2y+4(x+2\alpha(N-3)(N-4))\dot{y}\big) \\ &\quad\times\big[(N-3)y(\dot{y}+2(x+2\alpha(N-4)(N-5))\ddot{y}) \\ &\quad+2\dot{y}\big[(x-4\alpha(N-3)(N-4))\dot{y}+2x(x+2\alpha(N-3)(N-4))\ddot{y}\big]\big] \\ &\quad\times\frac{W^2}{2\alpha^2(N-3)^2(N-4)^2\big[6x\dot{y}+(N-5)y\big]^3}\,dx+2K_1\Bigg)\Bigg]^{\frac{1}{2}}, \end{aligned} \tag{43}$$

where K_1 is an integration constant. Equations (15) and (43) then yield the potential

$$\begin{aligned} Z &= \pm\Bigg[2\int\Bigg(x^2\big(-2y+4(x+2\alpha(N-3)(N-4))\dot{y}\big) \\ &\quad\times\big[(N-3)y(\dot{y}+2(x+2\alpha(N-4)(N-5))\ddot{y}) \\ &\quad+2\dot{y}\big[(x-4\alpha(N-3)(N-4))\dot{y}+2x(x+2\alpha(N-3)(N-4))\ddot{y}\big]\big] \\ &\quad\times\frac{W^2}{2\alpha^2(N-3)^2(N-4)^2\big[6x\dot{y}+(N-5)y\big]^3}\,dx+2K_1\Bigg)\Bigg]^{\frac{1}{2}}\frac{1}{W} \\ &\quad+\frac{(N-3)[2\alpha(N-4)(N-5)+x]y+2x(x+2\alpha(N-3)(N-4))\dot{y}}{2\alpha(N-3)(N-4)[6x\dot{y}+(N-5)y]}. \end{aligned} \tag{44}$$

The function Z, and consequently the metric potential $\lambda(r)$, are defined explicitly in terms of variables x and y. An analytic form for y must satisfy the constraint provided in Equation (41): we show that this equation can be integrated in general.

7.1. Condition $y-2(x+2\alpha(N-3)(N-4))\dot{y}=0$

From (41) we obtain

$$y-2(x+2\alpha(N-3)(N-4))\dot{y} = 0. \tag{45}$$

Equation (45) is a first order linear ordinary differential equation for which we can obtain the solution as

$$y = \tilde{Q}\sqrt{x+2\alpha(N-3)(N-4)}, \tag{46}$$

where $\tilde{Q}$ is an integration constant.

The solution of (44) for Z in Section 7.1 is given by

$$\begin{aligned} Z &= 1\pm\sqrt{2K_1}x\left[\frac{x+2\alpha(N-3)(N-4)}{(N-2)x+2\alpha(N-3)(N-4)(N-5)}\right]^{\frac{1}{3}} \\ &\quad+\frac{x}{2\alpha(N-3)(N-4)}. \end{aligned} \tag{47}$$

Therefore we have found a solution for Z in terms of elementary functions. We believe that the result (47) is new for $N\geq 5$.

7.2. Condition $(N-3)\dot{y}-2x\ddot{y}=0$

In this case, from (41), we obtain

$$(N-3)\dot{y}-2x\ddot{y} = 0, \tag{48}$$

which is a second order linear ordinary differential equation. The solution to (48) can be easily expressed by

$$y = \frac{2B_1 x^{\frac{N-1}{2}}}{N-1} + B_2, \tag{49}$$

where B_1 and B_2 are constants of integration.

Then the integrals in (44) can be evaluated for the form of y given in (49). Using MATHEMATICA we have the solution

$$\begin{aligned} Z = & \pm\Bigg[\Bigg[B_1(N-1)x^{\frac{N-5}{2}}\left[\frac{x^2(N-2)}{N-1} + 4\alpha(N-3)(N-4)x\right. \\ & +\frac{4\alpha^2(N-1)(N-3)^2(N-4)^2}{N-2} \\ & \left.-\frac{B_2(N-1)^2\sqrt{x}(x(N-2)+2\alpha(N-3)(N-4)(N-5))^2}{(N-2)(N-5)\left[8B_1(N-2)x^{\frac{N}{2}} + B_2(N-1)(N-5)\sqrt{x}\right]}\right]\Bigg] \\ & \times\frac{1}{2\alpha^2(N-3)^2(N-4)^2} + 2K_1\Bigg]^{\frac{1}{2}} \frac{x^{-\frac{N-5}{4}}}{\sqrt{8B_1(N-2) + B_2(N-1)(N-5)x^{-\frac{N-1}{2}}}} \\ & +\Bigg[(N-3)[2\alpha(N-4)(N-5)+x]\left(\frac{2B_1 x^{\frac{N-1}{2}}}{N-1} + B_2\right) \\ & +2B_1 x^{\frac{N-1}{2}}(x + 2\alpha(N-3)(N-4))\Bigg] \\ & \times\frac{1}{2\alpha(N-3)(N-4)\left[6B_1 x^{\frac{N-3}{2}} + \frac{2B_1(N-5)x^{\frac{N-1}{2}}}{N-1} + B_2(N-5)\right]}. \end{aligned} \tag{50}$$

The potential Z is presented in explicit form for all $N > 5$.

The case $N = 5$ has to be considered separately. With $N = 5$, the condition of pressure isotropy (12) with $y = \frac{2B_1 x^{\frac{N-1}{2}}}{N-1} + B_2$, gives the form

$$\begin{aligned} Z = & \frac{1}{3} \pm \left[\frac{x^2}{64\alpha^2} + \frac{(B_2)^2}{144\alpha^2(B_1)^2x^2} + \frac{x}{6\alpha} - \frac{B_2}{9B_1\alpha x} + 2K_1\right]^{\frac{1}{2}} \\ & + \frac{x}{8\alpha} + \frac{B_2}{12\alpha B_1 x}. \end{aligned} \tag{51}$$

Thus we have obtained a solution for the potential Z in terms of elementary functions.

In this class of models we are able to obtain two possible analytic forms for the function y in terms of elementary functions for all $N \geq 5$. The constraint equation in (41) is satisfied. The solution of y obtained from the first condition Section 7.1 is not contained in earlier models and the potential Z is represented by elementary functions. It is a new solution. In the second condition Section 7.2 we are able to obtain a result, with the use of the substitution (15), for the potential y similar to that found by Hansraj et al. [31] in five dimensions. In addition we have established that the condition of pressure isotropy can be integrated for $N > 5$ with the condition Section 7.2 to yield (50). It is observed that the potential Z in (51) contains algebraic functions of x and powers of the variable x. We believe that the new solutions arising from the conditions Sections 7.1 and 7.2 are due to the transformation (15) introduced in our analysis to transform the condition of pressure isotropy to canonical form. It is also important to note that these two classes of solutions exist only in EGB gravity. They do not have an Einstein limit as $\alpha \neq 0$.

In general the condition of pressure isotropy, in the context of general relativity, has been widely studied when the pressure is isotropic. Many families of exact solutions have been found over the years which help to investigate the physical properties in particular spacetimes and to provide insights into the geometry of stellar models. In spite of the many investigations that have been performed, the general solution of the condition of pressure isotropy equation is not known. Therefore many studies have been devoted to pursue the nature of exact solutions. There exist many algorithms that have been found to provide a systematic method to understand the nature of solutions that are possible. These algorithms do provide new insights into the behaviour of the model; however such studies provide only a limited insight into the behaviour of the gravitating system. In recent times strong evidence has become available indicating that the standard theory of gravitation, namely classical general relativity needs some modification. A geometric way of achieving this is to adapt the action of general relativity to include higher order curvature corrections from combinations of the Riemann tensor, the Ricci tensor and the Ricci scalar. A leading contender in what is now called modified gravity, is the class of Lovelock theories. The quadratic version in the Lovelock class is the EGB category of models which is quadratic in the action. Only a few exact solutions are known in quadratic EGB gravity, and fewer results are known in higher order Lovelock theories. It is important to generate new exact solutions in EGB gravity to understand the gravitational dynamics. The only systematic approach to study this problem was recently performed by Maharaj et al. [52]. In their approach a known exact solution is used to produce a new solution to the modified field equations. Other approaches possibly exist which may provide new insights, and can provide new models when the approach in [52] cannot be applied. It is therefore important to perform a study of the condition of pressure isotropy in EGB gravity in a general setting. The transformation used in this paper provides such a new approach to study the nature of solutions. The treatment of this paper shows that the condition of pressure isotropy can be brought to a simpler form using the transformation of Durgapal and Bannerji [54]. It turns out that the canonical form of pressure isotropy in EGB gravity is an Abelian differential equation. Some earlier results have shown that the relevant condition is an Abelian differential equation in particular spacetimes; our analysis in this paper shows that the canonical representation of the condition of pressure isotropy is always in the form of an Abelian differential equation. In this context see the treatment of Maharaj et al. [52]. Such equations are difficult to integrate. This explains the scarcity of exact solutions to the field equations in an EGB setting. A detailed study in Lovelock gravity theories is essential because of the physical importance of higher order gravity models.

8. Discussion

We have investigated static spherically symmetric models in a higher dimensional EGB gravity setting. The matter distribution considered is a perfect fluid with isotropic pressure. The EGB field equations for such a fluid distribution were generated for all spacetime dimensions. We showed that the condition of pressure isotropy is an Abelian differential equation of the second kind. A coordinate transformation was introduced to reduce this differential equation to the canonical form $w\dot{w} = F_1 w + F_0$. We were able to present a solution generating algorithm to this equation for all dimensions $N \geq 5$. A specific choice of the function y then leads to a functional form of w (and therefore Z). We illustrated that the specific functional form for $y = \sqrt{x}$ yields a new family of exact solutions for the potential Z presented by a sixth order polynomial for all dimensions N. The five dimensional limit $N = 5$ also yields a new solution, not contained in earlier treatments. We also found exceptional metrics when the canonical form $w\dot{w} = F_1 w + F_0$ does not apply. In addition setting $F_1 = 0$ yielded two different new classes of solutions with explicit functional forms of y and Z. We observed that the special case $N = 5$ (with $F_1 = 0$) yields results contained in earlier models. Other choices of y in (17) are possible that may lead to physically acceptable results.

We point out some avenues for future research. Firstly the assumption made in this paper is the Pascal principle. The stability of the Pascal principle has been studied in general relativity in the past. It is important that the stability of the condition of pressure isotropy in EGB gravity be considered in future research. Secondly it is important to build stellar models in relativistic astrophysics. This requires a study of the junction conditions at the stellar surface. The junction conditions in EGB gravity are different from general relativity as shown by Davis [58] in a seminal treatment. Thirdly exact solutions to the condition of pressure isotropy should be found in general Lovelock gravity theories, for example in third order Lovelock gravity. Our results hold in EGB gravity which is second order. New physical features will arise in other Lovelock gravity theories.

An important point needs to be highlighted. The analysis in this paper shows that general relativity is special; the condition of pressure isotropy is a linear differential equation in Z. In EGB gravity the condition of pressure isotropy is, in general, a nonlinear differential equation (an Abelian differential equation of the second kind) in Z. It is this feature of EGB gravity that allows for the new solution generating algorithm in Theorem 2 to arise. It would be interesting to investigate possible extensions of this result in general Lovelock theories and other modified gravity theories.

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Appendix A. Derivation of the EGB Field Equations

We provide an outline for the derivation of the EGB field equations (8) for arbitrary spacetime dimensions $N \geq 5$. It is not possible to generate the equations directly from software packages because of the presence of higher-order curvature terms. We have found curvature terms for $N = 5, 6, 7, 8$ and then induced the general forms for quantities related to curvature.

The nonvanishing Ricci components for the metric (5) read as

$$R^0{}_0 = e^{-2\lambda}\left[\nu'\lambda' - (\nu')^2 - \nu'' - \frac{(N-2)\nu'}{r}\right], \tag{A1a}$$

$$R^1{}_1 = e^{-2\lambda}\left[\nu'\lambda' - (\nu')^2 - \nu'' + \frac{(N-2)\lambda'}{r}\right], \tag{A1b}$$

$$R^2{}_2 = e^{-2\lambda}\left[\frac{\lambda'}{r} - \frac{\nu'}{r} - \frac{(N-3)}{r^2}\right] + \frac{N-3}{r^2}, \tag{A1c}$$

$$R^{N-1}{}_{N-1} = R^{N-2}{}_{N-2} = \cdots = R^2{}_2. \tag{A1d}$$

Using system (A1) the resulting Ricci scalar becomes

$$R = 2\left[\frac{(N-2)(N-3)}{2r^2} - e^{-2\lambda}\left(\nu'' + \nu'^2 - \lambda'\nu' - \frac{(N-2)\lambda'}{r} + \frac{(N-2)\nu'}{r} + \frac{(N-2)(N-3)}{2r^2}\right)\right]. \tag{A2}$$

Therefore the nonzero Einstein tensor components in N dimensions are generated as

$$G^0{}_0 = (N-2)\left[\frac{(N-3)e^{-2\lambda}}{2r^2} - \frac{e^{-2\lambda}\lambda'}{r} - \frac{(N-3)}{2r^2}\right], \tag{A3a}$$

$$G^1{}_1 = (N-2)\left[\frac{(N-3)e^{-2\lambda}}{2r^2} + \frac{e^{-2\lambda}\nu'}{r} - \frac{(N-3)}{2r^2}\right], \tag{A3b}$$

$$G^2{}_2 = e^{-2\lambda}\left[\nu'' + \nu'^2 + \frac{(N-3)\nu'}{r} - \frac{(N-3)\lambda'}{r} - \nu'\lambda' + \frac{(N-3)(N-4)}{2r^2}\right] - \frac{(N-3)(N-4)}{2r^2}, \tag{A3c}$$

$$G^{N-1}{}_{N-1} = G^{N-2}{}_{N-2} = \cdots = G^2{}_2. \tag{A3d}$$

The nonvanishing Gauss-Bonnet tensor components in N dimensions are expressed by

$$\begin{aligned}
H^0{}_0 &= \frac{2(N-2)}{r^4e^{4\lambda}}(N-3)(N-4)\left(e^{2\lambda}-1\right)\left(2r\lambda' + \frac{(N-5)\left(e^{2\lambda}-1\right)}{2}\right),\\
H^1{}_1 &= \frac{2(N-2)}{r^4e^{4\lambda}}(N-3)(N-4)\left(1-e^{2\lambda}\right)\left(2r\nu' - \frac{(N-5)\left(e^{2\lambda}-1\right)}{2}\right),\\
H^2{}_2 &= \frac{4}{r^2e^{2\lambda}}(N-3)(N-4)\left(-\nu''-\nu'^2+\nu'\lambda'\right)\\
&\quad + \frac{2(N-3)(N-4)}{r^2e^{4\lambda}}\left[-6\nu'\lambda' + 2\left(\nu''+\nu'^2\right)\right]\\
&\quad + 4(N-3)(N-4)\left[\frac{(N-5)}{r^3e^{4\lambda}}\left(1-e^{2\lambda}\right)\left(\nu'-\lambda'\right)\right]\\
&\quad + \frac{(N-3)(N-4)(N-5)(N-6)\left(e^{2\lambda}-1\right)^2}{r^4e^{4\lambda}},\\
H^{N-1}{}_{N-1} &= H^{N-2}{}_{N-2} = \cdots = H^2{}_2.
\end{aligned}$$

Then from (A3) and the above system we can easily compute $G^a{}_b - \frac{\alpha}{2}H^a{}_b$ which is the left hand side of (3).

The right hand side of (3) is described by

$$T^a{}_b = \mathrm{diag}(-\rho, p, p, ..., p). \tag{A4}$$

The EGB field equations (8) then follow.

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