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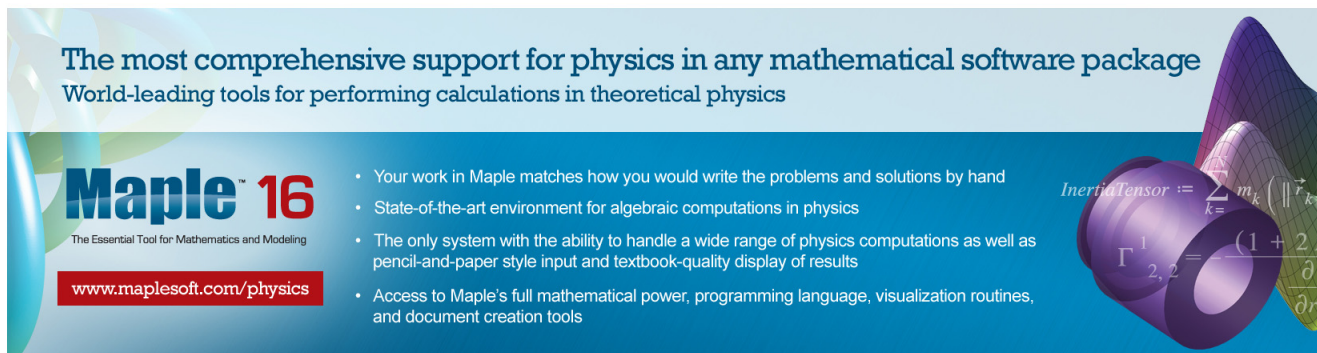
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Shearing radiative collapse with expansion and acceleration

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We investigate the behaviour of a relativistic spherically symmetric radiative star with an accelerating, expanding and shearing interior matter distribution in the presence of anisotropic pressures. The junction condition can be written in standard form in three cases: linear, Bernoulli, and Riccati equations. We can integrate the boundary condition in each case and three classes of new solutions are generated. For particular choices of the metric we investigate the physical properties and consider the limiting behaviour for large values of time. The causal temperature can also be found explicitly. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.3698286]

I. INTRODUCTION

The problem of radiative gravitational collapse was first investigated by Oppenheimer and Snyder.[1] Their interior spacetime is represented by a Friedmann-like solution for an isotropic homogeneous universe, and the exterior spacetime is described by the exterior Schwarzschild metric. The process of gravitational collapse is highly dissipative. Therefore heat flow in the interior of the star must be present, and taken into account so that the interior solution of the radiating star can match to the Vaidya[2] exterior metric at the boundary. The investigation of the gravitational behaviour of a collapsing star depends on the determination of the junction conditions matching the interior metric with the exterior Vaidya metric across the boundary of the star. Santos[3] formulated the junction conditions for a shear-free fluid distribution with isotropic pressures and made it possible to complete the model. His treatment paved the way to investigate physical features such as surface luminosity, dynamical stability, relaxation effects, and temperature profiles. Raychaudhuri,[4] showed that the slowest collapse arises in the case of shear-free fluid interiors. Kolassis *et al.*[5] assumed geodesic fluid trajectories when generating an exact model. Their model was generalised to include several new classes of solution in geodesic motion by Thirukkanesh and Maharaj.[6] In the past many investigations in radiating collapse have focussed on shear-free spacetimes with isotropic pressures (see the treatments of Herrera *et al.*,[7] Maharaj and Govender,[8] Herrera *et al.*,[9] and Misthry *et al.*[10]).

The next stage of development was to include shear in the model of a radiating star. Naidu *et al.*[11] included anisotropic pressures in the presence of shear for the interior spacetime and found simple exact solutions for geodesic fluid trajectories. This toy model was generalised by Rajah and Maharaj[12] by demonstrating solutions to a Riccati boundary equation governing the gravitational behaviour. The general situation requires a model which is expanding, accelerating, and shearing. Nogueira and Chan,[13] modelling shear viscosity and bulk viscosity, attempted such a study but found that they needed to utilise numerical techniques to make progress. Some recent progress has been made in finding exact models for Euclidean stars by Herrera and Santos[14] and Govender *et al.*[15] In Euclidean stars with shear the areal radius and proper radius are equal throughout the evolution of the radiating star. Our objective here is to show that it is possible to solve the relevant equations,

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for the general case, exactly in a systematic fashion. Our approach is the first analytic treatment to consider exact models with all the kinematical quantities present. We believe that these solutions will be helpful in studying physical features of a relativistic star in an astrophysical setting.

In this paper, we attempt to perform a systematic treatment of the governing equation at the boundary of the relativistic star with the interior consisting of a fluid which has nonzero acceleration, expansion, and shear. The junction condition is a nonlinear partial differential equation containing all three metric functions of spherical symmetry. In Sec. II, we derive the field equations and the junction conditions. In Sec. III, we give the boundary differential equation governing the gravitational behaviour of a radiating, shearing, and accelerating sphere. In Sec. IV, three new classes of exact solutions to the boundary condition are found in closed form. In Sec. V, we briefly investigate the physical features of the model generated and present the explicit form of the causal temperature for a particular choice of the metric functions. Some concluding remarks are made in Sec. VI.

II. THE MODEL

The most general form for the interior space time of a spherically symmetric collapsing star, which is expanding, accelerating, and shearing, is given by the line metric

$$ds^2 = -A^2 dt^2 + B^2 dr^2 + Y^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where A, B, and Y are in general functions of both the temporal coordinate t and the radial coordinate r. The existence of a fluid 4-velocity vector **u** enables us to introduce the kinematical quantities

$$\dot{u}^a = u^a{}_{;b}u^b, \quad \Theta = u^a{}_{;a}, \quad \sigma_{ab} = h_a{}^c h_b{}^d u_{(c;d)}, \tag{2}$$

where $h_{ab} = g_{ab} + u_a u_b$ ($h_{ab}u^a = 0$) is the symmetric projection tensor. The acceleration vector $\dot{u}^a$ ($\dot{u}^a u_a = 0$) represents the acceleration of the fluid particles relative to the congruences of **u**; the expansion scalar Θ measures the rate of increase of a volume of fluid element; the shear σ_{ab} ($\sigma_{ab}u^b = 0 = \sigma^a{}_a$) represents the tendency of a sphere to distort to an ellipsoid. For the comoving fluid 4-velocity $u^a = \frac{1}{A}\delta^a_0$ and the line element (1), the acceleration vector $\dot{u}^a$, the expansion scalar Θ and the magnitude of the shear scalar σ are given by

$$\dot{u}^a = \left(0, \frac{A'}{AB^2}, 0, 0\right), \tag{3a}$$

$$\Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + 2\frac{\dot{Y}}{Y}\right), \tag{3b}$$

$$\sigma = -\frac{1}{3A}\left(\frac{\dot{B}}{B} - \frac{\dot{Y}}{Y}\right), \tag{3c}$$

where primes and dots on the metric functions denote differentiation with respect to r and t, respectively. The energy momentum tensor for the interior matter distribution has the form

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab} + q_a u_b + q_b u_a + \pi_{ab}, \tag{4}$$

where ρ is the density of the fluid, p is the isotropic pressure, q_a is the heat flux vector, and π_{ab} is the stress tensor. The stress tensor can be expressed as

$$\pi_{ab} = (p_r - p_t)\left(n_a n_b - \frac{1}{3}h_{ab}\right), \tag{5}$$

where p_r is the radial pressure, p_t is the tangential pressure, and **n** is a unit radial vector given by $n^a = \frac{1}{B}\delta^a_1$. The isotropic pressure

$$p = \frac{1}{3}(p_r + 2p_t) \tag{6}$$

relates the radial pressure and the tangential pressure.

For the line element (1) and matter distribution (4) the coupled Einstein field equations become

$$\rho = \frac{2}{A^2}\frac{\dot{B}}{B}\frac{\dot{Y}}{Y} + \frac{1}{Y^2} + \frac{1}{A^2}\frac{\dot{Y}^2}{Y^2}$$
$$- \frac{1}{B^2}\left(2\frac{Y''}{Y} + \frac{Y'^2}{Y^2} - 2\frac{B'}{B}\frac{Y'}{Y}\right), \tag{7a}$$

$$p_r = \frac{1}{A^2}\left(-2\frac{\ddot{Y}}{Y} - \frac{\dot{Y}^2}{Y^2} + 2\frac{\dot{A}}{A}\frac{\dot{Y}}{Y}\right)$$
$$+ \frac{1}{B^2}\left(\frac{Y'^2}{Y^2} + 2\frac{A'}{A}\frac{Y'}{Y}\right) - \frac{1}{Y^2}, \tag{7b}$$

$$p_t = -\frac{1}{A^2}\left(\frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{Y}}{Y} - \frac{\dot{A}}{A}\frac{\dot{Y}}{Y} + \frac{\ddot{Y}}{Y}\right)$$
$$+ \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'}{A}\frac{B'}{B} + \frac{A'}{A}\frac{Y'}{Y} - \frac{B'}{B}\frac{Y'}{Y} + \frac{Y''}{Y}\right), \tag{7c}$$

$$q = -\frac{2}{AB^2}\left(-\frac{\dot{Y}'}{Y} + \frac{\dot{B}}{B}\frac{Y'}{Y} + \frac{A'}{A}\frac{\dot{Y}}{Y}\right), \tag{7d}$$

where the heat flux $q^a = (0, q, 0, 0)$ has only the nonvanishing radial component. A comprehensive treatment of the effects of anisotropy with heat flow in general relativity was carried out by Herrera *et al.*;[16] the first study with anisotropy appears to be in the treatment of Lemaitre.[17] The system of Eqs. (7a)–(7d) governs the general model when describing matter distributions with anisotropic pressures in the presence of heat flux for a spherically symmetric relativistic stellar object. For this model (7a)–(7d) describe the nonlinear gravitational interaction for a shearing matter distribution which is expanding and accelerating. From (7a)–(7d), we observe that if forms for the gravitational potentials A, B, and Y are known, then the expressions for the matter variables ρ, p_r, p_t, and q follow immediately. When the radial and tangential pressures are identical then $p_r = p_\perp$ which generates an additional nonlinear partial differential equation called the condition of pressure isotropy.

The Vaidya exterior spacetime[2] of a radiating star is given by

$$ds^2 = -\left(1 - \frac{2m(v)}{R}\right)dv^2 - 2dvdR + R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{8}$$

where $m(v)$ denotes the mass of the fluid as measured by an observer at infinity. The line element (8) represents coherent null radiation. The flow of the radiation is restricted to the radial direction relative to the hypersurface Σ, which represents the boundary of the star. The matching of the metric potentials and extrinsic curvature for the interior spacetime (1) and the exterior spacetime (8) produces junction conditions on the hypersurface Σ. These can be written as

$$A(R_\Sigma, t)dt = \left(1 - \frac{2m}{R_\Sigma} + 2\frac{dR_\Sigma}{dv}\right)^{\frac{1}{2}}dv, \tag{9a}$$

$$Y(R_\Sigma, t) = R_\Sigma(v), \tag{9b}$$

$$m(v)_\Sigma = \left[\frac{Y}{2}\left(1 + \frac{\dot{Y}^2}{A^2} - \frac{Y'^2}{B^2}\right)\right]_\Sigma, \tag{9c}$$

$$(p_r)_\Sigma = (qB)_\Sigma. \tag{9d}$$

The junction conditions (9a)–(9d) were first derived by Santos[3] for a shear-free radiating relativistic star. It is important to note the nonvanishing of the radial pressure at the boundary Σ. Thus, there is an additional differential equation (9d) which has to be satisfied together with the system of Einstein field equations (7a)–(7d). Junction conditions similar to (9d) are important in describing phenomena which arise in astrophysics. Di Prisco *et al.*[18] generated junction conditions relevant to spherical collapse with dissipation, in the presence of shear, for nonadiabatic charged fluids. Causal thermodynamics, in the context of the Israel-Stewart theory, was utilised by Herrera *et al.*[19] to study viscous dissipative gravitational collapse in both the streaming out and diffusion approximations.

III. THE BOUNDARY CONDITION

Substituting (7b) and (7d) in (9d) we obtain the boundary condition which has to be satisfied at the stellar surface,

$$2Y\ddot{Y} + \dot{Y}^2 - 2\left(\frac{\dot{A}}{A} + \frac{A'}{B}\right)Y\dot{Y} + 2\frac{A}{B}Y\dot{Y}'$$

$$-2\frac{A}{B^2}\left(A' + \dot{B}\right)YY' - \frac{A^2}{B^2}Y'^2 + A^2 = 0. \tag{10}$$

Equation (10) is the governing equation that determines the gravitational behaviour of the radiating anisotropic star with nonzero shear, acceleration, and expansion. It is clear that (10) is highly nonlinear; it is difficult to solve without making certain simplifying assumptions. Some exact solutions to (10) were found by Naidu *et al.*[11] and Rajah and Maharaj[12] for particles in geodesic motion ($\dot{u}^a = 0$) but expansion $\Theta \neq 0$ and shear $\sigma \neq 0$. Chan[20] considered the general case with $\dot{u}^a \neq 0$, $\Theta \neq 0$, and shear $\sigma \neq 0$ but no exact solutions were found. Instead the boundary condition was analysed numerically to study the physical features of the model, producing a final state where the star has radiated away mass during collapse. By assuming a relation between the metric functions B and Y for Euclidean stars Govender *et al.*[15] found particular models with shear.

Our intention is to solve (10) exactly without restricting the functions. For convenience we rewrite (10) in the following form:

$$\dot{B} - \left[\frac{\ddot{Y}}{AY'} + \frac{\dot{Y}^2}{2AYY'} - \frac{\dot{A}}{A^2}\frac{\dot{Y}}{Y'} + \frac{A}{2YY'}\right]B^2 - \left[\frac{\dot{Y}'}{Y'} - \frac{A'}{A}\frac{\dot{Y}}{Y'}\right]B + \left[A' + \frac{AY'}{2Y}\right] = 0. \tag{11}$$

In general (11) is a Riccati equation in the gravitational potential B. This Riccati equation can be solved in special cases.

IV. EXACT SOLUTIONS

The complexity and nonlinearity in (11) makes it difficult to solve in general. However, particular exact solutions can be found if we view (11) as a first order differential equation in the variable B and place restrictions on the bracketed expressions. We demonstrate this in the following three cases.

A. Linear equation

Note that Eq. (11) becomes a linear equation if we set

$$\frac{\ddot{Y}}{AY'} + \frac{\dot{Y}^2}{2AYY'} - \frac{\dot{A}}{A^2}\frac{\dot{Y}}{Y'} + \frac{A}{2YY'} = 0. \tag{12}$$

This equation can be written as

$$\dot{A} - \left[\frac{\ddot{Y}}{\dot{Y}} + \frac{\dot{Y}}{2Y}\right]A = \frac{A^3}{2Y\dot{Y}}, \tag{13}$$

which is a Bernoulli equation in the variable A. Even though Y is an arbitrary function, this equation can be integrated in general, and we have

$$A^2 = \frac{Y\dot{Y}^2}{h(r) - Y}, \tag{14}$$

where $h(r)$ is a function of integration. With the result (14), we find that (11) becomes

$$\dot{B} - \left[\frac{\dot{Y}'}{Y'} - \frac{A'}{A}\frac{\dot{Y}}{Y'}\right]B + \left[A' + \frac{AY'}{2Y}\right] = 0, \tag{15}$$

which is linear in B.

The bracketed expressions in (15) contain the functions A, Y, and their derivatives. In spite of this difficulty it is possible to solve (15) and obtain B in general. Therefore, the solution for the junction condition (11) can be given by

$$A = \sqrt{\frac{Y\dot{Y}^2}{h(r) - Y}}, \tag{16a}$$

$$B = Y'\exp\left(-\int\frac{A'\dot{Y}}{AY'}dt\right) \times$$
$$\left\{k(r) - \int\left[\left(\frac{A'}{Y'} + \frac{A}{2Y}\right)\exp\left(\int\frac{A'\dot{Y}}{AY'}dt\right)\right]dt\right\}, \tag{16b}$$

$$Y = Y(t, r), \tag{16c}$$

where $k(r)$ is a function of integration. We believe that (16a)–(16c) is a new solution to the boundary condition (10). Note that the gravitational potential $Y(t, r)$ is an arbitrary function in this class of solution. Once Y is specified then an explicit form for A is generated from (14) and the integrals in (16a)–(16c) can be evaluated. Consequently, explicit forms for the metric functions A, B, and Y can be found. The choice for Y should be made to provide a physically reasonable model.

B. Bernoulli equation

Observe that Eq. (11) reduces to a Bernoulli equation if we set

$$A' + \frac{AY'}{2Y} = 0. \tag{17}$$

Integrating this equation we get

$$Y = \frac{C_1(t)}{A^2}, \tag{18}$$

where $C_1(t)$ is a function of integration. Substituting (18) into (11) we obtain

$$\dot{B} - \left[\frac{3}{2}\frac{\dot{C}_1}{C_1} - 4\frac{\dot{A}}{A} + \frac{\dot{A}'}{A'}\right]B$$
$$= \left[\frac{7}{2}\frac{\dot{A}\dot{C}_1}{AA'C_1} - 5\frac{\dot{A}^2}{A^2A'} - \frac{\ddot{C}_1}{2C_1A'} + \frac{\ddot{A}}{AA'} - \frac{\dot{C}_1^2}{4C_1^2A'} - \frac{A^6}{4C_1^2A'}\right]B^2, \tag{19}$$

which is a Bernoulli equation in the variable B.

The coefficients in (19) contain the functions A, C_1, and their derivatives; however, it can be integrated in general. On integrating (19) we can write

$$B = \frac{A'C_1^{3/2}}{A^4[\int I\,dt + g(r)]}, \tag{20}$$

where $g(r)$ is a function of integration and for convenience we have defined

$$I = -\frac{7}{2}\frac{C_1^{1/2}\dot{A}\dot{C_1}}{A^5} + 5\frac{\dot{A}^2 C_1^{3/2}}{A^6} + \frac{\ddot{C_1}C_1^{1/2}}{2A^4} - \frac{\ddot{A}C_1^{3/2}}{A^5} + \frac{\dot{C_1}^2}{4C_1^{1/2}A^4} + \frac{A^2}{4C_1^{1/2}}. \tag{21}$$

Therefore, the functions

$$A = A(t, r), \tag{22a}$$

$$B = \frac{A' C_1^{3/2}}{A^4[\int I dt + g(r)]}, \tag{22b}$$

$$Y = \frac{C_1}{A^2}, \tag{22c}$$

satisfy the junction condition (11). The model (22) is an exact solution to the boundary condition (10). Note that the gravitational potential $A(t, r)$ is an arbitrary function in this class of solution. Once A is specified, together with the integration constants C_1, then an explicit form for I can be determined. Then the metric functions A, B, and Y can be expressed in closed form in terms of elementary or special functions. The choice for A should be made on physical grounds.

C. Inhomogeneous Riccati equation

Note that Eq. (11) has the form of an inhomogeneous Riccati equation if we set

$$\frac{\dot{Y}'}{Y'} - \frac{A'}{A}\frac{\dot{Y}}{Y'} = 0. \tag{23}$$

Integrating this equation we get

$$A = \dot{Y}\alpha(t), \tag{24}$$

where $\alpha(t)$ is a function of integration. In this case (11) becomes

$$\dot{B} = \left[\frac{\dot{Y}(1+\alpha^2)}{2\alpha Y Y'} - \frac{\dot{\alpha}}{\alpha^2 Y'}\right]B^2 - \left[\dot{Y}'\alpha + \frac{\dot{Y}Y'\alpha}{2Y}\right]. \tag{25}$$

This is an inhomogenous Riccati equation which is difficult to analyse in general. However, we shall show that it is possible to integrate this equation by placing restrictions on the functions α and Y.

If we take α to be a real constant and Y to be the separable function

$$Y(t, r) = K(r)C(t), \tag{26}$$

where $K(r)$ and $C(t)$ are arbitrary functions of r and t, respectively, then Eq. (25) becomes

$$\dot{B} = \frac{(1+\alpha^2)}{2\alpha K'}\frac{\dot{C}}{C^2}B^2 - \frac{3}{2}\alpha K'\dot{C}. \tag{27}$$

The Riccati equation (27) is not in standard form. Consequently, we introduce the transformation

$$B = wC \tag{28}$$

to obtain

$$\left[\frac{2\alpha K'}{(1+\alpha^2)w^2 - 2\alpha K'w - 3\alpha^2 K'^2}\right]\dot{w} = \frac{\dot{C}}{C}. \tag{29}$$

The advantage of the form (29) is that it is a separable equation in the variables w and C. Equation (29) can be integrated if the constant α is specified. To demonstrate a simple exact

solution we take $\alpha = -2$. Then (29) can be written as

$$\frac{\dot{w}}{(5w - 6K')(w + 2K')} = -\frac{1}{4K'}\frac{\dot{C}}{C}.$$ (30)

On integrating the above equation we obtain

$$w = \frac{2K'[3C^4 + f(r)]}{5C^4 - f(r)},$$ (31)

where $f(r)$ is a function of integration. The metric function B then follows since $B = wC$.

Therefore, we have generated a new solution to the inhomogenous Riccati equation (25). The form of the solution is given by

$$A = -2K\dot{C},$$ (32a)

$$B = \frac{2K'C[3C^4 + f(r)]}{5C^4 - f(r)},$$ (32b)

$$Y = KC.$$ (32c)

The form of the solution (32a)–(32c) is particularly simple and does not involve further integration. The functions C, K, and f are arbitrary; the physics of a specific model investigated will determine their explicit form. It is remarkable that the explicit solution (32a)–(32c) can be found for the boundary condition (10) in this case; Riccati equations are difficult to solve and only limited classes of solution are known to exist.

V. EXAMPLE

The simple forms of the solutions found this paper make it possible to study the physical behaviour of the model. In this section, we briefly consider the physical features of the solution generated in Sec. IV C. For the gravitational potentials obtained in (32a)–(32c), we take $C(t) = t^2$, $K(r) = r$, and $f(r) = k$, where k is a real constant. For these values the kinematical quantities become

$$\dot{u}^a = \left(0, \frac{[5\tilde{t} - 1]^2}{4r[3\tilde{t} + 1]^2\sqrt{k\tilde{t}}}, 0, 0\right),$$ (33a)

$$\Theta = \frac{[3 + 26\tilde{t} - 45\tilde{t}^2]}{2r[-1 + 2\tilde{t} + 15\tilde{t}^2](k\tilde{t})^{\frac{1}{4}}},$$ (33b)

$$\sigma = \frac{16\tilde{t}}{3r[1 - 2\tilde{t} - 15\tilde{t}^2](k\tilde{t})^{\frac{1}{4}}},$$ (33c)

where we have set $\tilde{t} = t^8/k$ for convenience. From (33a)–(33c) we observe that the acceleration $\dot{u}^a$, the expansion Θ and the magnitude of the shear scalar are nonzero. These quantities remain finite in the interior apart from the stellar centre. Also note that in the limiting case as $t \to \infty$ the acceleration $\dot{u}^a \to 0$ the shear scalar $\sigma \to 0$ and expansion $\Theta \to 0$. As the model evolves for large time the kinematical quantities grow progressively smaller. From the forms of $\dot{u}^a$ and Θ given above we observe that the acceleration decreases more rapidly than the expansion for large time.

The matter variables become

$$\rho = \frac{[-3 - 43\tilde{t} + 15\tilde{t}^2 + 95\tilde{t}^3]}{2r^2[3\tilde{t} + 1]^2[5\tilde{t} - 1]\sqrt{k\tilde{t}}},$$ (34a)

$$p_r = \frac{1}{4r^2\sqrt{k\tilde{t}}}\left[\frac{3[5\tilde{t} - 1]^2}{[3\tilde{t} + 1]^2} - 5\right],$$ (34b)

$$p_t = \frac{4[-\frac{13}{k} - 45\tilde{t} - 95\tilde{t}^2 + 25\tilde{t}^3]}{r^2 k[1 - 2\tilde{t} - 15\tilde{t}^2]^2}, \tag{34c}$$

$$q = \frac{[1 + 25\tilde{t} - 165\tilde{t}^2 + 75\tilde{t}^3]}{4r^2[3\tilde{t} + 1]^3(k\tilde{t})^{\frac{3}{4}}}. \tag{34d}$$

From (34a)–(34d) we observe that the energy density ρ, radial pressure p_r, tangential pressure p_t, and heat flux q are continuous in the stellar interior, apart from the centre. At later times as $t \to \infty$ we note that $q \to 0$ so that the heat flux is radiated away during the process of gravitational collapse. It is interesting to see that the energy density ρ, radial pressure p_r, the tangential pressure p_t, and the heat flux q are proportional to r^{-2}, and are decreasing functions as we approach the boundary of the star. The behaviour that $\rho \propto r^{-2}$ is of physical importance. It is interesting to observe that this property is also present in Newtonian isothermal spheres and relativistic isothermal cosmological models as pointed out by Saslaw *et al.*[21]

A qualitative analysis of the matter variables, energy conditions, and stability is difficult to achieve for the interior matter distribution. However, it is possible to generate graphical plots which indicate physical viability. In Figs. 1–3 we have plotted the energy density ρ, the radial pressure p_r, and the tangential pressure p_t. We observe that $\rho > 0$, $p_r > 0$, and $p_t > 0$. In addition, we have the behaviour $\rho' < 0$ and $p_r' < 0$ so that ρ and p are decreasing functions outwards from the centre to
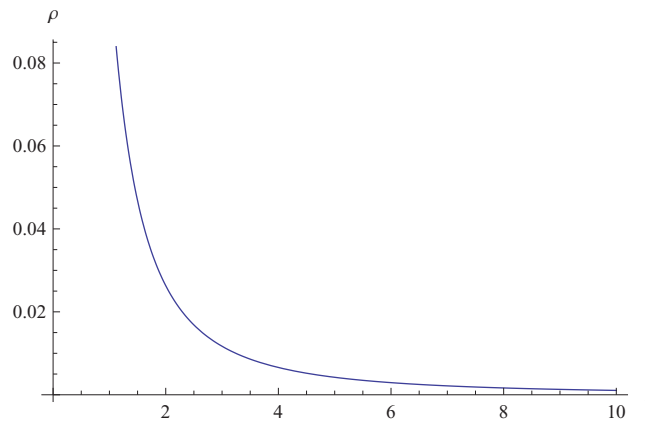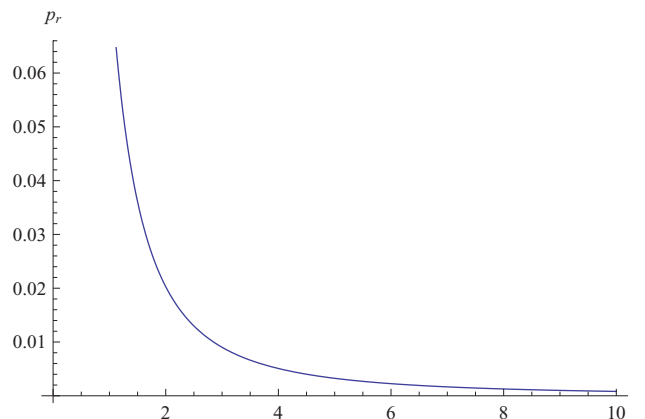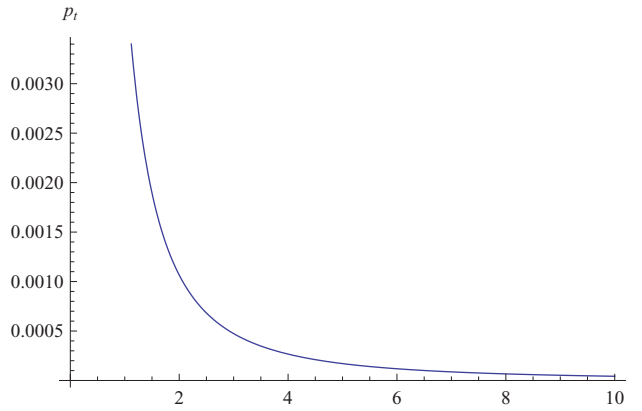

FIG. 1. Density.

FIG. 2. Radial pressure.

FIG. 3. Tangential pressure.

the stellar surface. For fixed values of the radial coordinate it is possible to plot the behaviour of

$$Z = (\rho + p_r)^2 - 4q^2, \tag{35}$$

$$Y = \rho - p_r - 2p_t + [(\rho + p_r)^2 - 4q^2]^{\frac{1}{2}}. \tag{36}$$

Typical behaviour of these quantities are represented in Figs. 4 and 5, respectively. These graphs show that $Z > 0$ and $Y > 0$. The behaviour exhibited in this physical analysis indicates that the weak, strong, and dominant conditions are satisfied in interior points away from the centre. Also note from Fig. 6 that the speed of sound is less than the speed of light so that causality is not violated.

Next, we briefly consider the relativistic effect of the causal temperature in our model. The Maxwell-Cattaneo heat transport equation, in the absence of rotation and viscous stresses is given by

$$\tau h_a^b \dot{q}_b + q_a = -\kappa \left(h_a^b \nabla_b T + T \dot{u}_a \right), \tag{37}$$

where τ is the relaxation time, κ is the thermal conductivity, $h_{ab} = g_{ab} + u_a u_b$ projects into the comoving rest space and T is the local causal temperature. Equation (37) reduces to the acausal Fourier heat transport equation when $\tau = 0$. The causal transport equation (37) can be written as

$$T(t, r) = -\frac{1}{\kappa A} \int \left[\tau(q\dot{B})B + AqB^2 \right] dr \tag{38}$$
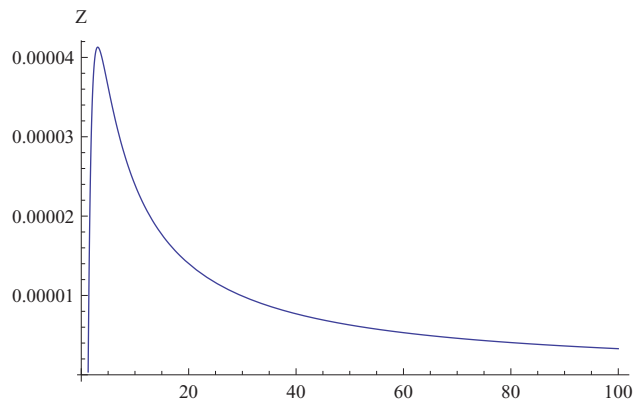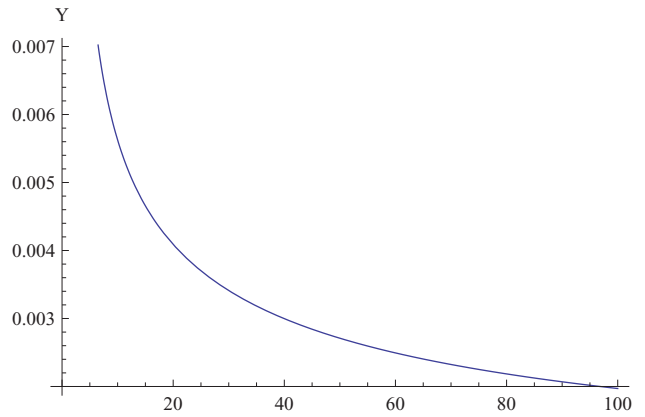

FIG. 4. $Z = (\rho + p_r)^2 - 4q^2$.

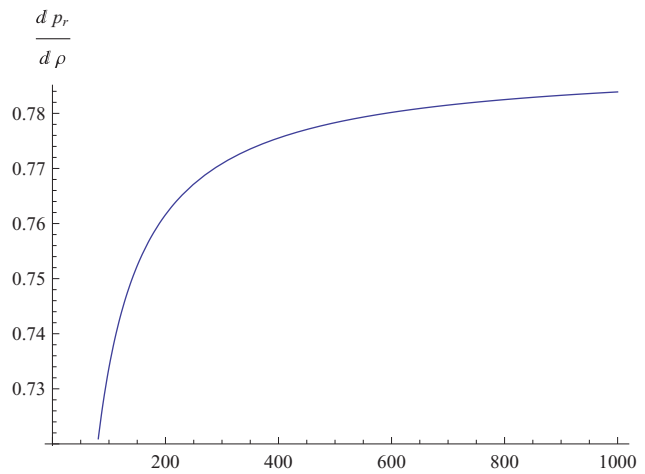FIG. 5. $Y = \rho - p_r - 2p_t + [(\rho + p_r)^2 - 4q^2]^{\frac{1}{2}}$.

FIG. 6. Sound speed.

for the metric (1). Martinez,[22] Govender *et al.*,[23] and Di Prisco *et al.*[24] have shown that the relaxation time τ has a major effect on the thermal evolution, particularly in the latter stages of collapse. Rajah and Maharaj[12] and Naidu *et al.*[11] showed that in the presence of shear stress, the relaxation time decreases as the collapse proceeds and the central temperature increases. For our case, (38) becomes

$$T(\tilde{t}, r) = \frac{\tau[-1 + 15\tilde{t} - 315\tilde{t}^2 + 45\tilde{t}^3]}{\kappa r^2[3\tilde{t} + 1]^2[5\tilde{t} - 1]\sqrt{k\tilde{t}}}$$

$$+ \frac{[1 + 30\tilde{t} - 15\tilde{t}^2(k\tilde{t})^{\frac{1}{4}}]\ln[r]}{\kappa r[-1 + 2\tilde{t} + 15\tilde{t}^2](k\tilde{t})^{\frac{1}{4}}} + h(t), \tag{39}$$

where $h(t)$ is a function of integration and we set τ and κ as constant. When $\tau = 0$, we can regain the acausal (Eckart) temperature from (39). It is possible to plot the causal and acausal temperatures against the radial coordinate. In Fig. 7 the temperature profiles are similar to the curves in Rajah and Maharaj.[12] The temperature is a decreasing function from the centre to the boundary of the star in both the causal and acausal curves. The inclusion of particle acceleration in our model may contribute to the more rapid decrease of temperature from the core to the boundary of the star. This may be applicable to phases of collapse where there is rapid expansion and cooling of the outer layers of the stellar fluid. As in the Rajah and Maharaj[12] model it is clear that the causal temperature is greater than the acausal temperature throughout the stellar interior.
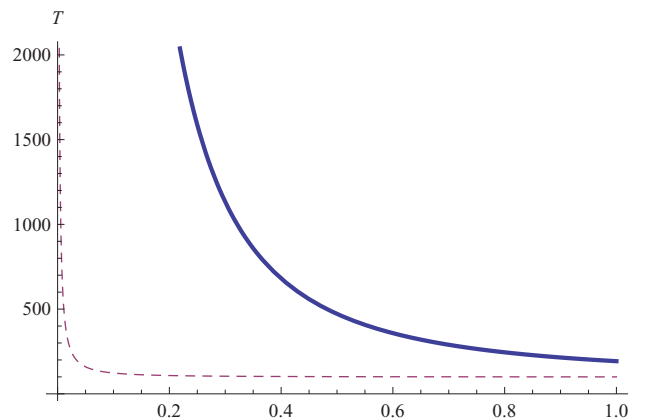
FIG. 7. Temperature: acausal (dashed) and causal (solid).

VI. DISCUSSION

In summary, we considered the general case of a spherically symmetric radiative star undergoing gravitational collapse when the interior spacetime consists of an accelerating, expanding, and shearing matter distribution. The junction condition is rewritten so that it can be considered as a first order equation in the potential $B(t, r)$. It is then possible to consider the junction condition as a standard differential equation: a linear equation, a Bernoulli equation, and a Riccati equation. The linear and Bernoulli equations are solved in general. The Riccati equation can only be solved for a particular value of the integration constant. Therefore three new classes of solutions to the boundary condition have been found. For a particular metric, corresponding to the inhomogenous Riccati equation, it is possible to obtain forms for the kinematical quantities and matter variables. It is then possible to indicate the behaviour of the model for large values of time.

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