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Full Length Research Paper

One-dimensional finite difference approach for sedimentation process in sand filled reservoirs

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There have been continuing efforts to decrease silt deposits due to interstitial blockage in sand-filled reservoirs. However, estimating weir height to allow for deposition of required grain sizes is difficult. This paper presents a numerical process-imitating model aimed at improving water storage potential in sand filled reservoirs. The technique uses a finite difference (FD) numerical model to numerically solve mass balance continuity equation built upon the conservative laws of solid-fluid mixture hydrodynamics. Present investigation shows that barrier height used in sand-filled reservoirs determines the grade quality of deposited sands. The predictions of the model compared with laboratory measurements show agreement between computational and experimental results. The study could provide economic and suitability data for increasing water supplies to a small community through determination of appropriate weir height that will increase subsurface water storage potential.

Key words: Groundwater, water supply, solid-fluid mixture, aquifer storage, numerical method.

INTRODUCTION

Reliable water source for domestic and livestock watering are much needed in remote rural communities under arid conditions. Provision of sufficient water under growing water demand is a major concern. However, regardless of these concerns, management of subsurface water storage with appropriate techniques will improve infiltration and aquifer recharge rates, and allow increased long-term average rates of groundwater storage, which will benefit a wide range of users. Alluvial deposit of seasonal rivers provide ideal medium for management of subsurface water storage. Previously, water loss due to surface evaporation, increasing siltation and dam safety issues has clearly showed the limitations of open surface storage. Water supplies from subsurface reservoirs such as water from dry river beds have been considered important part of integral solution to water-demand variability from persistent drought and climate change. Over the recent years, constructions of such are widely based on experience gained through trial-and-error and often not enhanced for water storage. The reservoir fill material absorbs water from periodic floods and yielding water to wells or infiltration galleries for abstraction.

Erratic stream flow makes groundwater bearing sediments in seasonal rivers essential source of water supplies in most dry areas. In rural areas, use of eroded sand deposit from catchments of a valley for groundwater development (Olufayo et al., 2009) has been documented. Apart from being sources of dependable water supply they provide acceptable protection against insect breeding and evaporation. However, sands delivered to the lower reach of a river will only be valuable storage medium for water storage only if the sands are of the right gradations. On the other hand, sediments will not accumulate without a natural or artificial barrier to slow the water (Van Haveren, 2004). Rural dwellers use barriers made of concrete to promote sediment depositions. However, the quality of sediments behind barrier is equally essential because deposition of silts is of little practical interest owing to its low specific yield. The barrier is not only to trap sediments but also to

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vary sizes of materials behind barrier. Wipplinger, 1958, observed fill material upstream of weir (artificial barrier) and inferred that, for efficient reservoir, the height above previous deposits should be in such a way that allows flow of silts over the barrier. Similarly, Gray and Leiser (1982) proposed sediment trapping in check dams but without consideration for material grades. The basic principle normally adopted in barrier designs for sand-filled reservoirs is to limit the size of stages so flow rates through the basin are enough to transport most of the fine sands over the dam crest. Presently there is no adequate study on evaluation of suitable weir heights which may mitigate silts production and thus increasing reservoir efficiencies. The inadequate study is often due to not enough field or laboratory investigations and numerical studies that may provide insight about the mechanisms of its operation. Although, silts are practically impossible to remove because with reduced floods, the rate of flow will always be low enough to deposit fine silts. However, later floods of higher extent would normally scour part or most of the deposited silt. Apart from silts filling up interstices of coarse material and reducing valuable void volumes for water storage, silts also impede recharge by slowing down infiltration rates causing near zero permeability occasionally. Much attention has been given to fluvial study in recent time. However, a quantitative understanding of alluvial bed form in sand filled reservoir and response to changes in governing conditions is still poorly understood. Multitudes of physical factors have been observed to influence bed form in channel as sediment transport rates vary in time and space (Madej et al., 2009). Extensive simulation of flow by other research work using various approaches, such as theoretical analysis, and laboratory experiments only give theoretical solution or empirical formulae in some instances for calculating flow (Jing et al., 2010) or sediment transport. Usually comparison between analytical and experimental data is not always satisfactory. This may be attributed to model simplifications adopted as some empirical correction might have been introduced to improve the comparison (Soni el al., 1980). Occasionally, empirical solutions based on site-specific observations and data may be useful for a particular site where the data were collected. Application of these solutions to any other sites should be treated with caution. In recent years, comprehensive laboratory experiments have been carried out to study sediment flow (Jing et al., 2010). Among them is a study performed by Renaat de Sutter and Krein (2001) which involved a series of sediment transport simulations during flood events using laboratory and field experiments to understand suspended load transport. On the other hand, many mobile bed developments still need further studies. Some of these are the correct evaluation of the liquid and solid phases, quantifying the solid discharge, estimating bed evolution (Schippa and Pavan, 2009) and sediment transport development involving soil detachment (Pal et al., 2001). Other studies examining mechanisms and conditions of sediment movement in fluid-grain mixtures in laboratory experiments include work of Madej et al. (2009), and van der Werf et al. (2009) which may find application in sand filled reservoirs. WU et al. (2003) explored fractional transport of sediment mixtures using new method based on Transport Capacity Fraction (TCF). The idea estimates the fractional transport rates for nonuniform sediment mixtures in sand-bed channels. However, different methods for predicting fractional sediment transport rates result in widely varying results that may also differ drastically from measurements. With increasing computational power, numerical methods have been more relevant and can be applied to simulation of flow. Compare with laboratory experiments, numerical approaches have advantages of non-intrusion and scaling (Jing et al., 2010). Numerical modeling from Schippa and Pavan (2009) reproduced bed evolution in channels of complex geometry of natural alluvial rivers. This was based on a conservative theory of one-dimensional shallow water equations which include impetus equation treatment as source term. The MacCormack explicit finite difference scheme was adopted in discretising governing equation. Wright and Parker (2005) also presented numerical modeling formulation for simulation of the longitudinal profile and bed sediment distribution in sand-bed rivers. Their study numerically investigated downstream decrease in bed slopes and downstream decrease in bed sediment diameters. Meanwhile, similar study of, Miglio et al. (2009) experimentally studied and simulated numerically aggradation and degradation in fluvial beds having uniform sediments. Numerical simulation is performed with the Double Order Approximation (DORA) model to solve one-dimensional shallow water equations governing a free surface; gradually varied unsteady flow on mobile beds. Comparison with experimental results enables assessing the deposition and erosion rates and determination of empirical law for the bed load discharge. Numerical simulation has proven to be a useful tool in addressing the plethora of complexity in sediment transport studies. However, several current available river morphology models are intended for specific applications such as solving sedimentation problems in river reach and reservoir siltation. Sediment transport studies involving sand filled reservoirs are rarely treated and few studies of this nature are reported in literature. Siltation process and designs of sand filled reservoir are not the same with classical reservoirs; so therefore, the physical parameters used to define the systems are not usually identical and there is a need for numerical study that can model sand filled reservoir to promote economical storage alternative.

In this paper, a process-based model for evaluating barrier heights is developed, and the reliability of the model is tested through application to the numerical reproduction in sand-filled reservoir experimental setup. In this study, we apply sediment routing through mass balance continuity equation in recirculating water supply system with rectangular cross-section. The Rubey-Watson equation for large and fine sediments in turbulent flow and unit stream power equation models were used to determine the grade quality of deposited sands. Finite different method (FDM) was employed to solve numerically the governing equation of the sediment laden flow. Well-documented laboratory experimental results were carried out to verify the computed results. The model can be used to determined ideal barrier height in the next incremental stage following complete siltation.

Governing equations and numerical scheme

In the proposed approach, the fluid-grain mixture is treated as fluid and lumps the suspended load and bed load together as the bed-material load. This eliminates the need to describe the boundary between the bed load and the bed material load. The model is based on the principle of conservation of mass and applied to the channels of 1D flow having non complex geometry. It is assumed that stream channels adjust towards equilibrium in which the ability of the channel to carry water and sediment is in balance with water and sediment delivered from upstream. Therefore, by neglecting the loss term, mass conservation for the liquid-grain mixture phase can be expressed as:

$$A\,\frac{\partial h}{\partial t} = I_s(t)in - Q_s(t)out \qquad (1)$$

in which I_s is the volumetric sediment-transport rate into basin, Q_s is the volumetric sediment –transport rate out of the basin, A is the surface area of sediments storage in the basin and h is the downstream barrier height and t represents time. Therefore, rearranging Equation (1) gives:

$$\frac{\partial h}{\partial t} = \frac{I_s(t)in - Q_s(t)out}{A} \qquad (2)$$

Sediment transport calculation

Sediment transport equation of Meyer-Peter and Müller (1948) is used for sediment inflow which is expressed as:

$$I_t^* = \begin{cases} \alpha_t\left(\varphi_s \tau^* - \tau_c^*\right)^{n_t}, & \tau^* > \tau_c^* \\ 0, & \tau^* \le \tau_c^* \end{cases} \qquad (3)$$

where τ_c^* denotes a critical Shields number for the onset of sediment motion, φ_s denotes the fraction of bed shear stress that is skin friction.

To complete the equation system Equation (2) two equation closures are used to estimate the particle settling rates in the basin and fractional particle discharge through the basin. The particle discharge can be calculated using several algebraic formulae reported in literature (Zeller and Fullerton, 1983, Finkner et al., 1989,). But, in the present method, Equation (4) has been selected (Dingman, 1984) based on easily measurable parameters from an experimental setup.

$$Q_s = 10^6 C_t \gamma_s Q \qquad (4)$$

where C_t represents total sediment concentration, γ_s is the particle weight density usually taken to be 2.65γ and Q is the water discharge. Suppose VS_o, U_*, υ, ω, V_{cr}, and d are unit stream power, which is a product of velocity and slope; shear velocity defined as $(gDS_o)^{1/2}$, where D is hydraulic depth), kinematic viscosity, fall velocity of sediment, critical velocity for erosion below which erosion will not occur and median particle diameter respectively. From Yang (1973), the total sediment concentration is then written as

$$C_t = J\left(\frac{VS_o}{\omega} - \frac{V_{cr}S_o}{\omega}\right)^K \qquad (5)$$

Yang (1976) relates empirically critical velocity for erosion, V_{cr}, with fall velocity by

$$\frac{V_{cr}S_o}{\omega} = 2.05S_o\text{, If } R_e \ge 70 \qquad (6)$$

$$\frac{V_{cr}S_o}{\omega} = \left(\frac{2.5}{0.434\ln R_e} + 0.66\right)S_o\text{, If } R_e < 7 \qquad (7)$$

where R_e is Reynolds number defined as $R_e = \frac{U_* d}{\upsilon}$ (8)

J and K are dimensionless empirical factors that depend on the characteristics of the flow and sediments defined as follows:

$$J = \frac{272.000}{R_p^{0.286}\left(\frac{U_*}{\omega}\right)^{0.457}} \qquad (9)$$

$$K = 1.799 - 0.178\ln R_p - 0.136\ln\left(\frac{U_*}{\omega}\right) \qquad (10)$$

In which Rp is the particle Reynolds number,

$$R_p = \frac{\omega dp}{\mu} \qquad (11)$$

On the other hand, the Rubey-Watson law is used to compute settling velocity as Watson (1969) modified stoke's law for small and large particles in turbulent flow. This is expressed as:

$$\omega = \frac{[3.48\mu^2 + 0.088\rho g(\rho_s - \rho)d^3]^{1/2} - 1.87\mu}{0.265\rho d} \quad (12)$$

where μ is the dynamic viscosity measured in kg/m-sec, ρ is the density of water in kg/m^3, ρ_s is the density of particle in kg/m^3 and d is the mean particle diameter in metre (m).

Hyperbolic nonlinear solution of Equation (2) is provided by the application of simply numerical scheme of finite difference method. This achieved a reasonable order of accuracy in short temporal step. Numerical solution of Equation (2) was based on one-side biased differencing of finite difference and used in discretization of the governing equation to evaluate differential terms of Equation (2) (Edsberg, 2008). Thus approximation of spatial grid node, *i,* is given by:

$$\left.\frac{\partial h}{\partial t}\right|_i \approx \frac{h_{i+1} - h_i}{\Delta t} = \frac{I_{si}(t)in - Q_{si}(t)out}{A_i} \quad (13)$$

where h_i represents the initial height of barrier, h_{i+1} is the final height, h_{i+1}-h_i is the change in height, ΔH, Δt is the temporal step size, A_i is the surface area at temporal grid level i, $I_{Si}in$ is sediment- transport rate in temporal grid level i, and $Q_{Si}out$ is sediment- transport out temporal grid node i. Therefore, mass continuity Equation (13) of sediment routing for 1D flow crest predictor can be expressed in compact form as follows:

$$\frac{\Delta H}{\Delta t} = \frac{I_{si}(t)in - Q_{si}(t)out}{A_i} \quad (14)$$

From the known values of τ_i^* the sediment transport rate I_t^* can be computed from Equation (3). Once the conservation equation is properly written for any particular problem, we generally proceed by substituting appropriate alternative expressions for the various terms and applying the laws of algebra, Dingman (1984).The relationship for the settling velocity, particle diameter of Watson (1969) which links particle diameter with the fall velocity for particle of silt size and larger, and unit stream power (Yang, 1976) is then substituted in the computation. However, it should be noted that particle shape affects the value of drag coefficient used in deriving it and hence the fall velocity and particle diameter (Dingman, 1984). The formulation of the mass conservation equation is written and taking the derivative with respect to time of each term in expression Eq. (2) for a given region of space giving the governing ordinary differential equation. To simplify the problem and organise the associated data and information the modeling protocol was structured as shown in Figure 1.

EXPERIMENTAL DESIGN

Alluvial materials for experiments

The sand materials used for the study comprises of mixture of grain sizes collected from conventional catchment of sand filed reservoir. At the early stage of all experiments, the sand samples collected were screened off grain sizes more than 6.7 mm. The mixed sediments had a natural grading with D_{50} = 0.1 mm, D_{75} = 2.0 mm, D_{25} = 0.52 mm, and D_{max} = 6.7 mm. While the subscript is the cumulative percentile of the particle-size distribution as shown in Table 1.

Experimental data

The physical experiments were conducted to study sediment sorting and verify model estimates, as a direct effect of varying barrier heights in an alluvial channel under steady flow conditions. A small laboratory flume was used, to which a barrier was placed downstream to simulate a silting basin. The flume was made of rectangular glass-sided material of dimensions 2 m x 30 x 30 cm (Figure 2). The integral set-up of the flume consists of a tilting device, recirculating water supply system and non-recirculating sediment-feed. Sediment was not recirculated to simulate the condition of imposed sediment load from catchment slope (Olufayo et al., 2010). The channel bed of the flume was roughened with non-cohesive 2.36 mm sand grains spread evenly and carefully glued to the surface to simulate an impermeable channel bed. The bed initial slope was set at 0.28%, a typical field condition for sand filled reservoirs, and downstream elevation was kept constant with a weir. A predetermined amount of clear water was introduced into the flume and non-uniform sediment fed through hopper-feeder arrangement at a constant feed rate of 54 g/s using belt conveyor. This was allowed to wash down over time until equilibrium slope was reached. The main characteristics of the experimental set-up are provided in Table (2). For the analysis presented here; Olufayo et al. (2010) experimental runs were used. The runs which were performed with three different weir heights and water discharge range from 6.3 to 8.1 m^3/h.

Development of sand filled reservoir considered three phases which have distinctive mode of sedimentation process. The initial stage of development results in substantial sediments trapping by barrier wall. Here the amount and size of trapped particles depend on height of barrier. Hidden effects of smaller particle by bigger ones usually take place along the bed. As the flow fully developed, during the second stage, sediment trapping in submerged barrier conditions takes effect. At this stage, particles trapping depend on height of barrier as well as other factors such as, particle density, fall velocity and hydraulic conditions. The settling rates of sediment particles deposited in the basin were estimated using the modified Stoke's law of Rubby-Watson (1969). The last stage was the re-suspension effect of sediment leading to downstream winnowing and sorting of sediments.

Numerical analysis

Boundary and initial conditions

In this simulation, subcritical flow was assumed. We consider a

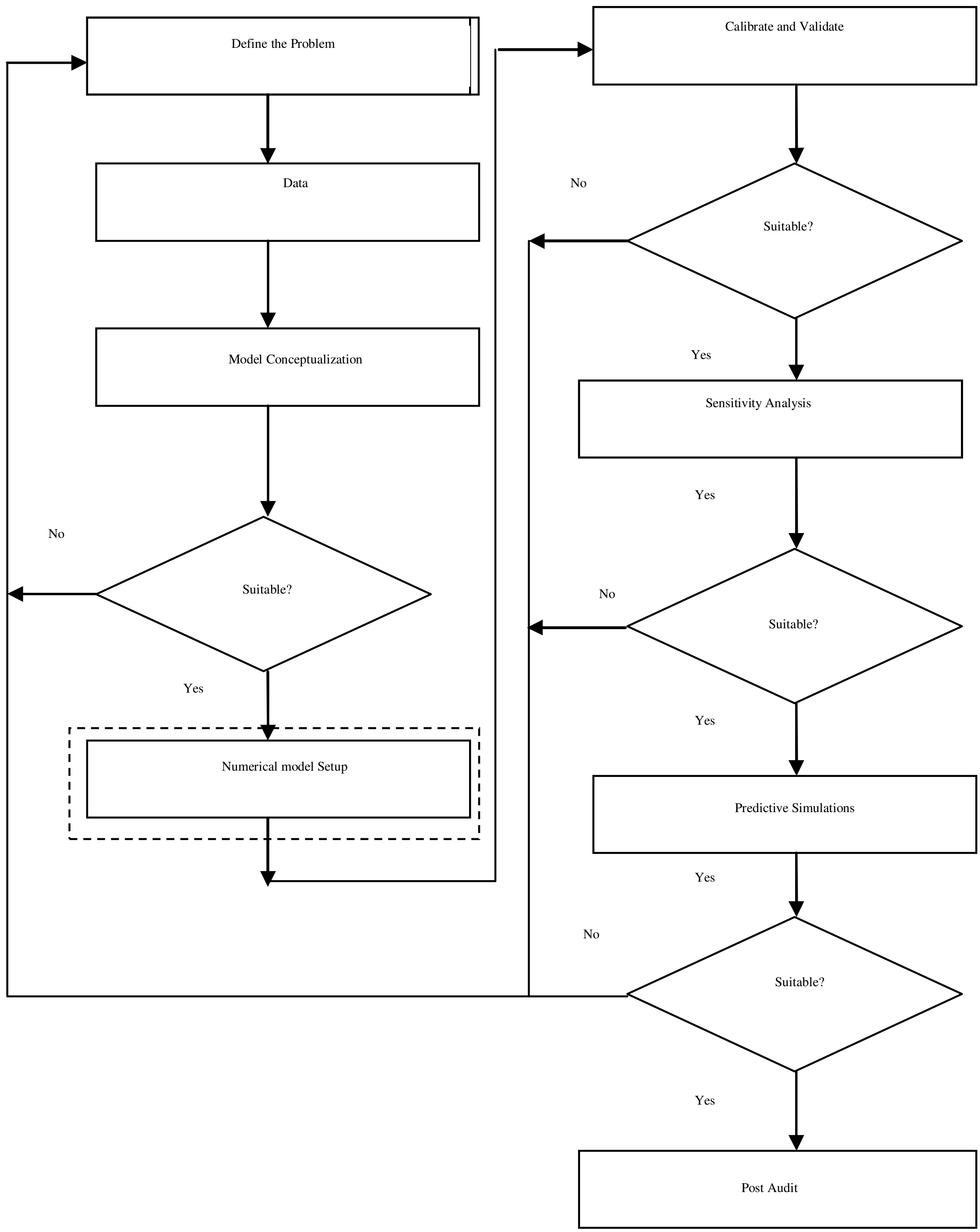

Figure 1. Modeling protocol.

Table 1. Representative size classes of sediment feed

Size class (mm)	Mean grain size (mm)	Fraction (%)	Cumulative %
< 0.062	-	-	-
0.062 - 0.125	0.075	2	2
0.125 - 0.25	0.15	8	10
0.25 - 0.5	0.3625	21	31
0.5 - 1.0	0.6	31	62
1.0 - 2.0	1.18	12	74
1.0 - 4.0	2.18	25	99
4.0 - 8.0	4.75	1	100

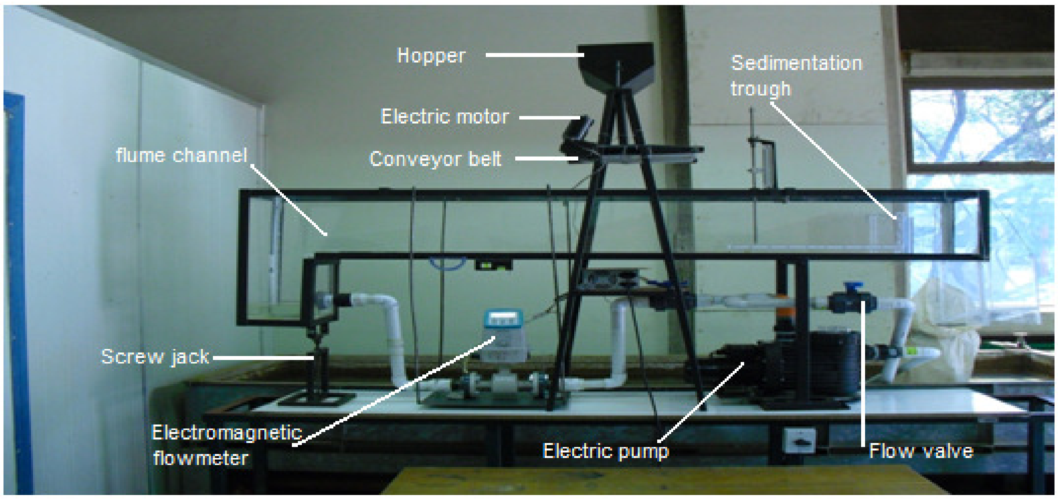

Figure 2. Picture of experimental apparatus.

Table 2. Characteristic of the experimental set-up data from Olufayo et al. (2010).

	Run 7	Run 5	Run 3
Q (m^3/h)	6.7	7.2	8.1
H (mm)	10	15	20
D (mm)	20	26.6	28
Fr	0.75	0.71	0.39

silting basin 450 mm length with a slope of 0.28%. The channel was rectangular with a fixed width of 0.3 m and the surface area is fixed. At the upstream boundary the sediment feed was set at 54 g/s and at the downstream end the initial condition was without a barrier. The time step is set at 20 mins.

RESULTS AND DISCUSSION

The numerical scheme was applied to set of experimental runs of sediment laden flow. Figure 2 shows the computed and observed bed sorting for various barrier heights of 10, 15, and 20 mm at different flow rates. The distribution profiles were reproduced with reasonable accuracy, although the agreement between measurements and computation is lower at 10 mm barrier height, in which sediment transport value is overestimated. When no barrier exists, the channel bed experiences pure degradation akin to Figure 3a. Flow retardation by 15 and 20 mm barriers produced a region of increase flow depth (backwater) upslope of the barrier in which sediment depositions varied with flow rates and height of weirs. The increasing heights of barrier in cases of Figure 3a, b and c were reflected in decreasing d_{50} for both observed and computed values. The sediment profile distribution is found to affect free surface profile. In particular, the free surface profile is noticeably lower in the upstream reach where bed aggradation occurs. During these processes, a saturated bed layer is formed which filled the pore of sediment as sediment is added

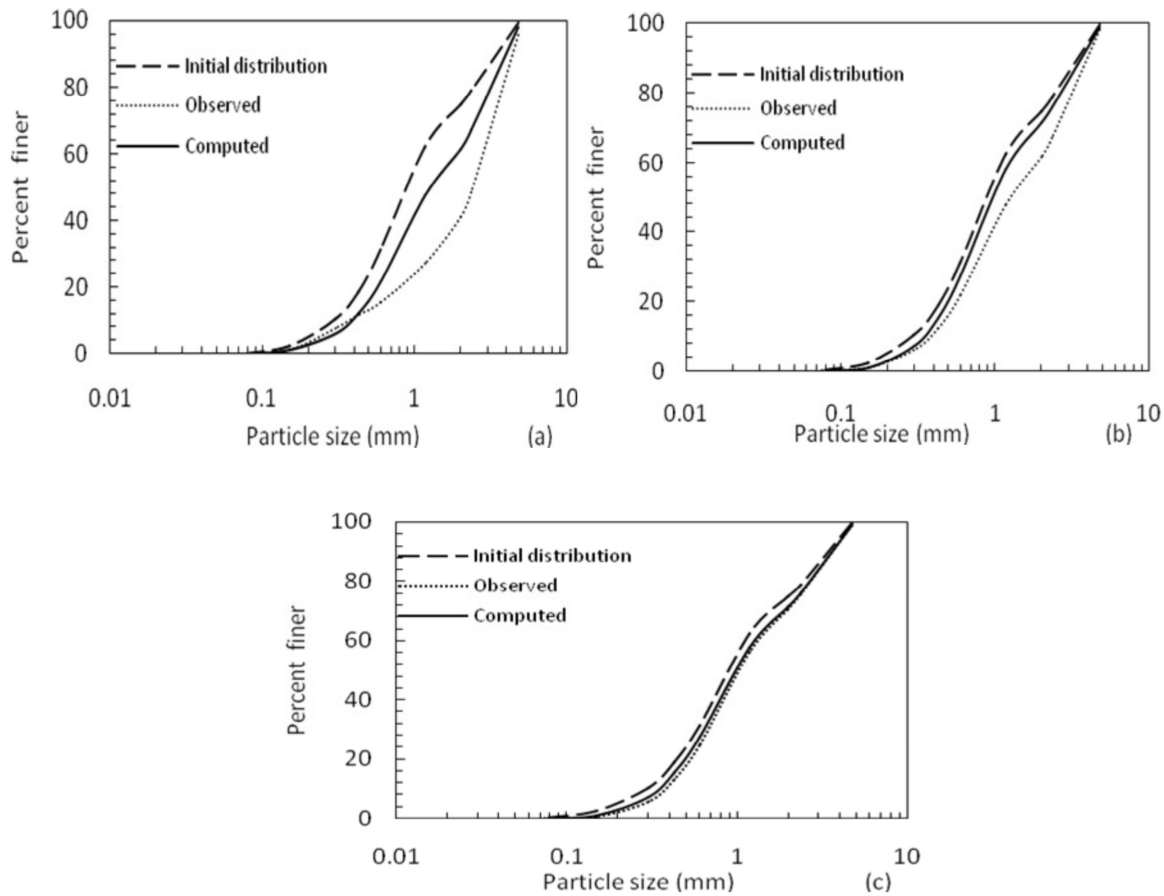

Figure 3. Bed sorting for Run # at weir height: (a) Run 7, 10 mm ($Q = 6.7\ m^3/h$); (b) Run 5, 15 mm ($Q = 7.2\ m^3/s$); and (c) Run 3, 15 mm ($Q = 8.1\ m^3/s$).

into the channel.

Sensitivity analysis

Generally, deposition or erosion in sediment transport is typically in non-equilibrium conditions. Selection of design discharge should be made after considering general physical, temporal characteristics and any other applicable hydrodynamic factors to determine the desired outcome. Thus a test run calculation was carried out to determine effect of range of discharges and inflow sediments in sensitivity analysis. A range of discharges as well as sediment inflows were used to run the calculations. A sensitivity analysis showed less variability in macropores of sand deposits at lower discharge a rate which markedly reduces water storage efficiency of the sand and increases silt deposits. This might give field-scale explanation for the often observed phenomenon of the small transfer rate of silts at low discharge. Figure 3a predicted grain distribution at low discharge reasonably well and agreed with experimental observations.

Conclusions

An experimental and numerical investigation on barrier phased elevation of crest in sand filled reservoirs was presented. A numerical model based on the solution of sediment routing through mass balance continuity equation was developed adopting finite different method scheme. The comparison between the numerical results and the laboratory measurements produced reasonable agreement. Sand filled reservoir sediment deposition and erosion depend on hydraulic parameters and height of barrier. However, bed sorting requires that bed composition be tracked for the percentage of each particle size class (Yang and Simoes, 2008). While this experimental model was not made after any prototype, this model is fully useful for the purpose of understanding essential qualities of deposition in sand filled reservoirs. The results of this study can be adopted for developing stepping construction of sand filled reservoirs to increase storage potential. However, further investigations are necessary to consider protection effects of fine sand particle and consolidation.

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